# The Asset Durability Premium

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#### Abstract

This paper studies how the durability of assets affects the cross-section of stock returns. More durable assets incur lowers frictionless user costs but are more "expensive", in the sense that they need more down payments making them hard to finance. In recessions, firms become more financially constrained and prefer "cheaper" less durable assets. As a result, the price of less durable assets is less procyclical and therefore less risky than that of durable assets. We provide strong empirical evidence to support this prediction. Among financially constrained stocks, firms with higher asset durability earn average returns about 5% higher than firms with lower asset durability. We develop a general equilibrium model with heterogeneous firms and collateral constraints to quantitatively account for such a positive asset durability premium.

JEL Codes: E2, E3, G12

**Keywords:** Durability; financial constraints; collateral, cross-section of stock returns

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## 1 Introduction

Durability is an essential feature of capital, and varies dramatically across types of assets. How does asset durability affect firms' equity risks and, in turn, the cost of capital? Rampini (2019) argues that asset durability significantly affects financing. In particular, more durable assets incur lower frictionless user costs but are more "expensive", in the sense that they need higher down payments making more durable assets hard to finance. In this paper, we build this insight into a canonical macroeconomic model with collateral constraints, and demonstrate that asset durability have profound implications on the risk profile on the asset side of firms' balance sheets, exactly through the impact of asset durability on the debt financing on the liability side.

A common prediction of a macro finance model with financial frictions is that financial constraints exacerbate economic downturns because they are more binding in bad time. In recessions, firms become more financially constrained and collectively prefer "cheaper" less durable assets that require less down payments. This creates a general equilibrium effect that the price of less durable assets is less procyclical and therefore less risky than that of durable assets. In sum, our theory predicts that less durable assets are less risky than more durable assets. We evaluate this mechanism through the lens of the cross-section of equity returns. In particular, our theory suggests that a firm holding a larger fraction of less durable assets commands a lower expected return, since less durable assets provide a hedge against the aggregate risks, especially in recessions when firms become more financially constrained.

To examine the empirical relationship between asset durability and expected returns, we first construct a measure of firm's asset durability. Asset durability of capital can be measured in two ways, either by modeling with geometric depreciation rates or with a finite service life, as in Rampini (2019). Our paper measures a firm's asset durability as the value-weighted average of the durability of the different types of assets owned by the firm.

Consistent with the theoretical prediction in our model, Our empirical study focuses on financially constrained firms. We construct five portfolios univariate sorted on firms' durability relative to firms' industry peers using the U.S. data on publicly traded firms. We show that the asset durability return spread, that is, the returns of a long high durability firms and short low durability firms portfolio among the financially constrained firms is statistically significant. Our empirical finding documents that the spread between the highest durability quintile portfolio and the lowest durability quintile portfolio is on average close to 4-7% per annum within the subset of financially constrained firms. We call the asset durability premium as the difference in average portfolio returns between the highest and

lowest portfolio sorted by the asset durability measure. A high-minus-low strategy based on the asset durability spread delivers an annualized Sharpe ratio of 0.59, comparable to that of the market portfolio. Moreover, according the asset pricing test shown in Section 6.2, the alphas remain significant even after controlling for Fama and French (2015) five factors or Hou, Xue, and Zhang (2015) (HXZ hereafter) q-factors, respectively. The evidence on the durability spread strongly supports our theoretical prediction that the durable capital is more risky and therefore earn a higher expected return than the non-durable capital.

We also empirically review the ability of firm-level durability to predict the cross-sectional stock returns using monthly Fama and MacBeth (1973) regressions. This analysis allows us to control for an extensive list of firm characteristics that predict stock returns. The slope coefficient associated with the firm's lagged durability is both economically and statistically significant. To be concrete, in the baseline specification in which we also control for the financial leverage of the firm, a one-unit standard deviation increase in the firm's durability is associated with an increase of 2.13% in firms' expected (future) stock return. For the robustness, we verify that the positive durability-return relation is not driven by other known predictors which are seemingly correlated with the durability measure.

To quantify the effect of asset durability on the cross-section of expected returns, we develop a general equilibrium model with heterogeneous firms and financial constraints. As in Kiyotaki and Moore (1997) and Gertler and Kiyotaki (2010), lending contracts can not be fully enforced and therefore require collateral. In our model, assets with different levels of asset durability are traded, and firms with higher financing needs but low net worth endogenously acquire less durable assets. This is because, as in Rampini (2019), a durable capital incurs a lower frictionless user cost but is costly with a higher upfront down payment and, therefore, hard to finance. In the economic downturns, firms become more financially constrained and prefer cheaper less durable capital collectively and in turn creates a general equilibrium price effect. In particular, firms with high productivity and low net worth face higher financing needs in equilibrium and tend to acquire cheaper assets (i.e., less durable assets with lower down payments). As a result, the price of less durable capital is less procyclical and, therefore, less risky than that of durable capital. In the constrained efficient allocation in our model, the heterogeneity in productivity and net worth translates into the heterogeneity in asset durability across firm assets. In this setup, we show that, at the aggregate level, more durable capital requires higher expected returns in equilibrium, and, in the cross-section, firms with high asset durability earn high risk premia.

In our quantitative analysis, we show that our model, when calibrated to match the conventional macroeconomic quantity dynamics and asset pricing moments, is able to generate

significant asset durability spread. As consistent with the data, firms with higher asset durability exhibit higher financial leverages. Quantitatively, our model matches the empirical relationship between asset durability, leverage, and expected returns in the data reasonably well.

On the empirical side, we further provide empirical evidence that directly support model implications. First, we document that the price of capital with higher durability exhibits higher sensitivities to the aggregate macroeconomic shocks. Second, we show that high asset durability firms have significant higher cash flow betas with respect to the aggregate TFP and GDP growth shocks. Third, we further follow the standard empirical procedure to estimate stochastic discount factor using the generalized method of moments (GMM), and show that the aggregate TFP and GDP growth shocks are significantly positively priced among durability-sorted portfolios. Firms with high durability are more positively exposed to these aggregate shocks and, therefore, demand for higher expected returns, consistent with our model interpretation.

#### 1.1 Related literature

Our paper builds on the corporate finance literature that emphasizes the importance of collateral for firms' capital structure decisions. Albuquerque and Hopenhayn (2004) study dynamic financing with limited commitment, Rampini and Viswanathan (2010, 2013) develop a joint theory of capital structure and risk management based on firms' asset collateralizability. Schmid (2008) considers the quantitative implications of dynamic financing with collateral constraints. Nikolov et al. (2018) studies the quantitative implications of various sources of financial frictions on firms' financing decisions, including the collateral constraint. Falato et al. (2013) provide empirical evidence for the link between asset collateralizability and leverage in aggregate time series and in the cross section. Our paper departs from the above literature in three important dimensions: first, we explicitly study firms' optimal asset acquisition decision among assets with different durability under the context of a collateral constraint, as in Rampini (2019). However, different from Rampini (2019), we bring an asset durability decision into a general equilibrium framework, take aggregate shocks into accounts, and then study the asset pricing implications of such a decision on the asset side of firms' balance sheets through the lens of the cross-sectional stock returns.

Our study builds on the large macroeconomics literature studying the role of credit market frictions in generating fluctuations across the business cycle (see Quadrini (2011) and Brunnermeier et al. (2012) for extensive reviews). The papers that are most related to ours are

those emphasizing the importance of borrowing constraints and contract enforcements, such as Kiyotaki and Moore (1997, 2012), Gertler and Kiyotaki (2010), He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014), and Elenev et al. (2018). Gomes et al. (2015) studies the asset pricing implications of credit market frictions in a production economy. We allow firms to optimally choose their asset durability, and study the implications of durable versus less durable capital on the cross-section of expected returns.

Our paper belongs to the literature of production-based asset pricing, for which Kogan and Papanikolaou (2012) provide an excellent survey. From the methodological point of view, our general equilibrium model allows for a cross section of firms with heterogeneous productivity and is related to previous work including Gomes et al. (2003), Gârleanu et al. (2012), Ai and Kiku (2013), and Kogan et al. (2017). Compared to the above papers, our model incorporates financial frictions and study their asset pricing implications. In this regard, our paper is closest related to Ai, Li, Li, and Schlag (2019) and Li and Tsou (2019), which both use a similar model framework and aggregation technique to study stock returns and the asset collateralizability and leasing versus secure lending, respectively. Ai, Li, Li, and Schlag (2019) shows that more collateralizable assets provide an insurance against aggregate shocks, because these assets help relax the collateral constraint, especially in recessions when the financial constraint becomes more binding.

Our paper is related to a recent literature on the duration premium in the cross-section. Papers, including Gonçalves (2019), Gormsen and Lazarus (2019) and Chen and Li (2018), show that firms with longer cash flow duration earn a lower average return than those with longer cash flow duration. Our paper is consistent with this evidence. In our model, other things being equal, firms that experienced a history of positive productivity shocks have a internal cash flow and optimally choose to obtain higher asset durability. Therefore, in the model, a history of high productivity shocks is associated with higher asset durability, higher ROE and but shorter cash flow duration. As shown in Table C.1, this feature of our model is consistent with the pattern in the data. In particular, higher asset durability firms display shorter Dechow et al. (2004) cash flow duration but high expected return, in line with the short cash flow premium documented in the above papers.

Our paper is also connected to the broader literature linking investment to the cross-section of expected returns. Zhang (2005) provides an investment-based explanation for the value premium. Li (2011) and Lin (2012) focus on the relationship between R&D investment and expected stock returns. Eisfeldt and Papanikolaou (2013) develop a model of organizational capital and expected returns. Belo, Lin, and Yang (2018) study implications of equity financing frictions on the cross-section of stock returns. Tuzel (2010) documents a

positive relation between firms' real estate holding and expected returns, and she proposes an adjustment cost explanation. Our paper focuses on a broader definition of asset durability, in which real estate is one particular kind of durable capital. Moreover, we propose a complementary financial constraint explanation. In the data, we find the asset durability premium is more significant among the financially constrained firms, which directly supports our model mechanism.

The rest of our paper is organized as follows. We summarize our empirical results on the relationship between asset durability and expected returns in Section 2. We introduce a general equilibrium model with collateral constraints in Section 3 and analysis the asset pricing implications in Section 4. In Section 5, we provide a quantitative analysis of our model. Section 6 provides supporting evidence of the model. Section 7 concludes. Details on data construction are delegated to the Appendix B. In Appendix C, we further provide some additional empirical evidence to establish the robustness.

## 2 Empirical Facts

This section provides some cross-sectional and aggregate evidence that highlight the asset durability as an important determinant of the cross-section of stock returns, especially for for financially constrained firms.

## 2.1 Measuring Asset Durability

To empirically examine the link between asset durability and expected returns and test our theoretical prediction, we need to construct a separate measure of asset durability with respect to physical assets (i.e., equipment, structures) and intangible assets (i.e., intellectual property and product). We measure an asset's durability as its service life by calculating the reciprocal of the asset's depreciation rate.

We construct the measure of asset durability using the Bureau of Economic Analysis (BEA) fixed asset table with non-residential detailed estimates for implied rates of depreciation and net capital stocks at fixed cost (hereafter referred to as the "BEA table"). The table breaks down depreciation rate on equipment, structures, and intellectual property and

<sup>&</sup>lt;sup>1</sup>Our data is provided by the Bureau of Economic Analysis (BEA) fixed asset table with non-residential detailed estimates for implied rates of depreciation and net capital stocks at fixed cost. This table breaks down implied rates of depreciation and net capital stocks into a variety of asset categories for a broad cross-section of industries.

product by 72 assets for 63 industries<sup>2</sup>, covering virtually all economic sectors in the United States.<sup>3</sup>

### Constructing the Industry- and Firm-level Asset Durability Measure

Given the BEA table with implied rates of depreciation, the durability of asset h employed by industry j in year t is computed as asset h's service life (i.e., the reciprocal of asset h's depreciation rate). We value-weight the asset-level durability across the 71 assets (equipment and structures) in the BEA table to obtain an industry-level asset durability index:

Asset 
$$Durability_{j,t}^K = \sum_{h=1}^{71} \bar{w}_{h,j,t} \times Asset \ Durability \ Score_{h,j,t}^K,$$
 (1)

where  $Asset\ Durability_{j,t}^K$  is a measure of asset durability for industry j in year t,  $\bar{w}_{h,j,t}$  represents industry j's capital stocks on asset h divided by its total capital stocks in year t from the BEA table, and  $Asset\ Durability\ Score_{h,j,t}^K$  is the durability score of asset h employed by industry j in year t. The resulting asset durability index represents a relative asset durability ranking of each industry's asset composition of tangible assets. On the other hand, we compute the asset durability of the intellectual property and product,  $Asset\ Durability_{j,t}^H$ , as the the reciprocal of industry j's depreciation rate in year t.

Further, we construct a firm-level measure of asset durability with respect to tangible and intangible assets as the value-weighted average of industry-level asset durability indices across business segments in which the firm operates:

$$Asset \ Durability_{i,t}^{K} = \sum_{j=1}^{n_{i,t}} \widetilde{w}_{i,j,t} \times Asset \ Durability_{j,t}^{K},$$

$$Asset \ Durability_{i,t}^{H} = \sum_{j=1}^{n_{i,t}} \widetilde{w}_{i,j,t} \times Asset \ Durability_{j,t}^{H},$$

$$(2)$$

where  $Asset\ Durability_{i,t}^K$  ( $Asset\ Durability_{i,t}^H$ ) is firm i's asset durability of tangible (intangible) capital,  $n_{i,t}$  is the number of industry segments, and  $\widetilde{w}_{i,j,t}$  is industry segment j's sales divided by the total sales for firm i in year t, and  $Asset\ Durability_{i,t}^K$  ( $Asset\ Durability_{i,t}^H$ )

<sup>&</sup>lt;sup>2</sup>We do not include detailed assets of the intellectual property and product because of missing data issue. Therefore, we consider the depreciation rate of the intellectual property and product at industry-level. Land is not included in the BEA non-residential asset categories. We assume land has infinite durability across industries.

<sup>&</sup>lt;sup>3</sup>The industry classification employed by the BEA is based on the 1997 North American Industry Classification System (NAICS). Therefore, we match the 63 BEA industries with Compustat firms using NAICS code.

<sup>&</sup>lt;sup>4</sup>In this paper, we use the terms "intellectual property and product" and "intangible" interchangeably.

is the asset dutiability of industry j in year t for the type-K (type-H) computed as equation (1).

Now we obtain firm i's asset durability of equipment and structures and that of intellectual property and product, respectively, and value-weight these two types of asset durability by their capital stocks, which refer to firm i's tangible capital PPEGT<sub>i,t</sub> and intangible capital INTAN<sub>i,t</sub> in year t, respectively, where  $w_{i,t}$  denotes firm i's relative weight of these two types of capital at time t.<sup>5</sup>

$$Asset \ Durability_{i,t} = w_{i,t} \times Asset \ Durability_{i,t}^K + (1 - w_{i,t}) \times Asset \ Durability_{i,t}^H. \tag{3}$$

In the main empirical analysis, we employ this firm-level measure, which is likely to provide more refined across-firm variation in asset durability than the industry-level one.<sup>6</sup> Due to the availability of the asset durability measure interacting with the U.S. data on publicly traded firms, our main analysis is then performed for the 1978 to 2016 period.

## 2.2 Asset Durability and Financial Constraints

Consistent with Rampini (2019), our model predict financial constraint is critical for firms to determine the composition of durable and less durable capital. With the firm level asset durability measure, we provide a first evidence that financial constraint is an important determinant for firms' asset durability decision, which supports both Rampini (2019) and our theoretical prediction.

In this subsection, we show that a firm's asset durability is increasing in its financial constraints. The asset durability increases in financial constraint since the capacity of external financing is declining. The empirical implication is that measures of financial constraint (i.e., non-dividend payment dummy<sup>7</sup>, SA index, WW index) should be negatively related to the asset durability. Moreover, to the extent that profitability contributes to internal funds, profitability should be positively related to the asset durability. Therefore, we examine these empirical predictions as follows.

#### [Place Table 1 about here]

The financial variables that we use are motivated by the empirical predictions of our mode,

<sup>&</sup>lt;sup>5</sup>Details in the measurement of intangibles refer to Ai, Li, Li, and Schlag (2019).

<sup>&</sup>lt;sup>6</sup>Our asset durability measure is robust to the measure constructed by using depreciation expenditure in Compustat.

<sup>&</sup>lt;sup>7</sup>In contrast to dividend payment dummy (DIV), non-dividend payment dummy (Non-Div) is whether a firm pays no dividend.

as well as by existing literature. We expect to find negative coefficients on non-dividend payment dummy, SA index, WW index, and a positive coefficient on profitability. As our model shows in later sections, variables that indicate that a firm is financially constrained, places a high value on internal fund, and, therefore, endogenously choose "cheaper" less durable assets, which is consistent with the negative correlation of a firm's financial constraint with its optimal decision for high durable assets.

Specification 1-4 of Table 1 reports the results of a univariate regression for each of the financial constraint or profitability, and specification 5-7 reports the results for a multivariate regression controlling for other fundamentals. Non-dividend dummy is significantly negatively related to asset durability both univariate and multivariate specification, which suggests that payout policy seems to be a direct measure of the value of internal funds. Such a negative relation to asset durability remains robust when we replace the non-dividend payment dummy by alternative financial constraint measures. Likewise, other financial constraint measure, SA and WW index, are also significantly negative related to asset durability, which is consistent with our theory that constrained firms prefer less durable assets and tend to hold larger internal funds to insure future negative aggregate shocks. Taking all together, results in Table 1 motivate us to shift our attention to financially constrained firms and further investigate the asset pricing implications in the following sections.

## 2.3 Asset Durability and Leverage

In Table 2, we construct the firm-level durability measure and report summary statistics of asset durability and book leverage for the aggregate and the cross-sectional firms in Compustat.

### [Place Table 2 about here]

Panel A reports the statistics of the financially constrained firm group versus its unconstrained counterpart. The constraint is measured by the dividend payment dummy (Farre-Mensa and Ljungqvist (2016), DIV hereafter).<sup>8</sup> Panel A presents two salient observations. First, the average of asset durability among financially constrained firms (12.66) is significantly lower than that of the unconstrained firms (16.54); that is to say, financially constrained firms use capital with higher durability (lower depreciation rate). Second, the average book leverage of constrained firms (0.24) is lower than that of unconstrained counterpart (0.33).

<sup>&</sup>lt;sup>8</sup>We tried other financial constrained measures, including SA index, credit rating, and WW index. These four proxies show consistent results empirically.

In panel B, we further sort financially constrained firms in the Compustat into five quintiles based on their asset durability relative to their industry peers as NAICS 3-digit industry classifications, and report firm characteristics across five quintiles. First, we observe a large dispersion in the average asset durability (depreciation), ranging from 7.69 (0.19) in the lowest quintile (Quintile L) to a ratio as much as 18.00 (0.11) in the highest quintile (Quintile H). Second, the book leverage is upward sloping from the lowest to the highest asset durability sorted portfolio. From these findings in Table 2, we recognize that asset durability can be a critical determinant of external financing activities for the constrained group, and that it is the first-order determinant of the capital structure on the firms' liability side. In the next section, we will present evidence to show that asset durability also plays an important role on firms' asset side, as reflected by equity returns across firms with heterogenous asset durability.

## 2.4 Asset Durability and Expected Returns

We zoom in on on the subset of financially constrained firms, consistent with our theory that firms' asset valuations contain a non-zero Lagrangian multiplier component. We consider four alternative measures for the degree to which a firm is financially constrained: the dividend payment dummy (Farre-Mensa and Ljungqvist (2016), DIV hereafter), the Size-Age index (Hadlock and Pierce (2010), SA index hereafter), the credit rating (Farre-Mensa and Ljungqvist (2016), Rating hereafter), and the Whited-Wu index (Whited and Wu (2006), Hennessy and Whited (2007), WW index hereafter). A firm is classified as a financially constrained firm if its dividend payment is zero, if its credit rating is missing, or if its WW (SA) index is higher than the median in a given year.

To investigate the link between asset durability and future stock returns in the cross-section, we construct five portfolios sorted on a firms's current asset durability and report the portfolio's post-formation average stock returns. We construct the durability at an annual frequency as described in Section 2.1. We focus on annual rebalancing (as opposed to monthly rebalancing) to minimize transaction costs of the investment strategy. At the end of June of year t from 1978 to 2017, we rank firms by asset durability relative to their industry peers and construct portfolios as follows. Specifically, we sort all firms with positive asset durability in year t-1 into five groups from low to high within the corresponding NAICS 3-digit industries. As a result, we have industry-specific breaking points for quintile portfolios for each June. We then assign all firms with positive asset durability in year t-1 into these portfolios. Thus, the low (high) portfolio contains firms with the lowest (highest) asset durability in each industry. To examine the asset durability-return relation, we form a high-minus-low portfolio that

takes a long position in the high durability portfolio and a short position in the low asset durability portfolio.

After forming the six portfolios (from low to high and high-minus-low), we calculate the value-weighted monthly returns on these portfolios over the next twelve months (July in year t to June in year t+1). To compute the portfolio-level average excess stock return in each period, we weight each firm in the portfolio by the size of its market capitalization at the time of portfolio formation. This weighting procedure enables us to give relatively more weight to the large firms in the economy and hence it minimizes the effect of the very small firms (and hence potentially difficult to trade) on the results (also note that we drop firms with fewer than 1 million assets or sales from the sample to further decrease the influence of the small firms on our results).

### [Place Table 3 about here]

In Panel A (Panel B) of Table 3, the top row presents the annualized average excess stock returns (E[R]- $R_f$ , in excess of the risk free-rate), standard deviations, and Sharpe ratios of the five portfolios sorted on asset durability. With Table 3, we show that, consistent with our model, a firm's asset durability forecasts stock returns. Firms with currently low asset durability earn subsequently lower returns, on average, than firms with currently high asset durability.

Table 3 presents the result that the average excess returns on the first five portfolios increase with asset durability. In the first panel of Panel A, the average excess return for firms with high asset durability (Portfolio H) is higher on an annualized basis than that with low asset durability (Portfolio L). Moreover, the average excess return on the high-minus-low portfolio is 6.93% with statistical significance with a t-value of 2.86 and a Sharpe ratio 0.59. The difference in returns is economically large and statistically significant. We find the positive asset durability-return relation and statistical significance on the long-short portfolio. We call the return spread of a long-short high-minus-low (Portfolio H-L) strategy the durability premium. The premium is robust with respect to the alternative measure of financial constraint, as can be seen from the second to the fourth panel. In Panel B, we find that the average excess returns on five portfolios increase with durability; however, the long-short portfolio return is amount to 1.44% and statistically insignificant.

Overall, the evidence on the asset durability spread among financially constrained firms strongly supports our theoretical prediction that more durable assets are more risky and, therefore, are expected to earn higher expected returns. In the following section, we develop a general equilibrium model with heterogeneous firms and financial constraints to formalize

the above intuition and to quantitatively account for the positive asset durability premium.

## 3 A General Equilibrium Model

In this section, we describe the ingredients of our quantitative model of the asset durability spread. The aggregate aspect of the model is intended to follow standard macro models with collateral constraints such as Kiyotaki and Moore (1997) and Gertler and Kiyotaki (2010). We allow for heterogeneity in the durability of assets as in Rampini (2019). The key additional elements in the construction of our theory are idiosyncratic productivity shocks and firm entry and exit. These features allow us to generate quantitatively plausible firm dynamics in order to study the implications of asset durability for the cross-section of equity returns.

### 3.1 Households

Time is infinite and discrete. The representative household consists of a continuum of workers and a continuum of entrepreneurs. Workers (entrepreneurs) receive their labor (capital) incomes every period and submit them to the planner of the household, who make decisions for consumption for all members of the household. Entrepreneurs and workers make their financial decisions separately.<sup>9</sup>

The household ranks the utility of consumption plans according to the following recursive preference as in Epstein and Zin (1989):

$$U_{t} = \left\{ (1 - \beta) C_{t}^{1 - \frac{1}{\psi}} + \beta (E_{t}[U_{t+1}^{1 - \gamma}])^{\frac{1 - \frac{1}{\psi}}{1 - \gamma}} \right\}^{\frac{1}{1 - \frac{1}{\psi}}},$$

where  $\beta$  is the time discount rate,  $\psi$  is the intertemporal elasticity of substitution, and  $\gamma$  is the relative risk aversion. As we will show later in the paper, together with the endogenous growth and long run risk, the recursive preference in our model generates a volatile pricing kernel and a sizable equity premium as in Bansal and Yaron (2004).

In every period t, the household purchases the amount  $B_{i,t}$  of risk-free bonds from entrepreneur i, from which she will receive  $B_{i,t}R_{f,t+1}$  next period, where  $R_{f,t+1}$  denotes the risk-free interest rate from period t to t+1. In addition, the household receives capital in-

<sup>&</sup>lt;sup>9</sup>According to Gertler and Kiyotaki (2010), we make the assumption that household members make joint decisions on their consumption to avoid the need to keep the distribution of entrepreneur income as an extra state variable.

come  $\Pi_{i,t}$  from entrepreneur i. We assume that the labor market is frictionless, and therefore the labor income from worker members is  $W_tL_t$ . The household budget constraint at time t can therefore be written as

$$C_t + \int B_{i,t}di = W_t L_t + R_{f,t} \int B_{i,t-1}di + \int \Pi_{i,t}di.$$

Let  $M_{t+1}$  denote the stochastic discount factor implied by household optimization. Under recursive utility, the stochastic discount factor denotes as,  $M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}}{E_t[U_{t-1}^{1-\gamma}]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}$ , and the optimality of the intertemporal saving decisions implies that the risk-free interest rate must satisfy

$$E_t[M_{t+1}]R_{f,t+1} = 1.$$

## 3.2 Entrepreneurs

There is a continuum of entrepreneurs in our economy indexed by  $i \in [0, 1]$ . Entrepreneurs are agents operating productive ideas. An entrepreneur who starts at time 0 draws an idea with initial productivity  $\bar{z}$  and begins the operation with an initial net worth  $N_0$ . Under our convention,  $N_0$  is also the total net worth of all entrepreneurs at time 0 because the total measure of all entrepreneurs is normalized to one.

Let  $N_{i,t}$  denote entrepreneur i's net worth at time t, and let  $B_{i,t}$  denote the total amount of risk-free bond the entrepreneur issues to the household at time t. Then the time-t budget constraint for the entrepreneur is given as

$$q_{d,t}K_{i,t+1}^d + q_{nd,t}K_{i,t+1}^{nd} = N_{i,t} + B_{i,t}. (4)$$

In equation (4) we assume that two types of capital,  $K^d$  and  $K^{nd}$ , differ in their asset durability. That is, the former capital is more durable, while the latter capital is less durable. For the brevity of reference, we denote these two types of capital with a superscript d for durable and nd for non-durable, respectively. These two types of capital depreciate at geometric depreciation rates  $\delta_d < \delta_{nd}$  each period, with  $\delta_h \in (0,1)$ , for  $h \in \{d,nd\}$ . We use  $q_{d,t}$  and  $q_{nd,t}$  to denote their prices at time t, respectively.  $K^d_{i,t+1}$  and  $K^{nd}_{i,t+1}$  are the amount of capital that entrepreneur i purchases at time t, which can be used for production over the period from t to t+1. We assume that the entrepreneur only has access to risk-free borrowing contracts, i.e., we do not allow for state-contingent debt. At time t, the entrepreneur is assumed to have an opportunity to default on his contract and abscond with

 $1 - \theta$  of both types of capital. Because lenders can retrieve a  $\theta$  fraction of the type-j capital upon default, borrowing is limited by

$$B_{i,t} \le \theta \sum_{h \in \{d, nd\}} q_{h,t} K_{i,t+1}^h. \tag{5}$$

Note that in the collateral constraint (5) we assume both types of capital have the same collateralizability parameter  $\theta$ . This is an assumption we maintain in order to single out the effect of asset durability. In Rampini (2019) and in reality, durability could also simultaneously affect collateralizability. For instance, in Rampini (2019), he assumes a collateral constraint of the form  $B_{i,t} \leq \theta \sum_{h \in \{d,nd\}} q_{h,t} K_{i,t+1}^h (1 - \delta_h)$ , in which the effective collateralizability

ability becomes  $\theta(1-\delta_h)$  and more durable capital (i.e. lower  $\delta_h$ ) is more collateralizable.

In our paper, there is a critical distinction between the durability and the collateralizability of an asset. According to Ai et al. (2019), an asset with a higher collateralizability lowers the riskiness of assets, as an insurance to aggregate shocks by relaxing the financing constraint. However, unlike that of the asset collateralizability, the mechanism of asset durability affects not only the duration of asset but also the price of the underlying asset. In our model, an asset with a longer duration is more expensive, incurs a higher down payment, therefore, is more difficult to finance, as highlighted in Rampini (2019). Such the mechanism implies that the price of more durable assets is more sensitive to aggregate shocks; that is to say, assets with longer duration embody higher riskiness than those with shorter duration. In the quantitative part of our paper, we also consider a variation of the model with Rampini (2019) type of collateral constraint in which durability simultaneously affects colateralizability, we show that quantitatively the net effect of the asset durability is to raise the riskiness of firm assets. In summary, our model in this paper explicitly distinguishes asset durability from asset collateralizability and predicts that asset durability could increase the riskiness of the underlying asset by impeding financing. Moreover, we show that our theoretical prediction is empirically plausible in terms of testable implications on the cross-section of equity returns.

From time t to t+1, the productivity of entrepreneur i evolves according to the law of motion

$$z_{i,t+1} = z_{i,t}e^{\varepsilon_{i,t+1}},\tag{6}$$

where  $\varepsilon_{i,t+1}$  is a Gaussian shock with mean  $\mu_{\varepsilon}$  and variance  $\sigma_{\varepsilon}^2$ , assumed to be i.i.d. across agents i and over time. We use  $\pi\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}^d, K_{i,t+1}^{nd}\right)$  to denote entrepreneur i's equilibrium profit at time t+1, where  $\bar{A}_{t+1}$  is aggregate productivity in period t+1, and  $z_{i,t+1}$ 

denotes entrepreneur i's idiosyncratic productivity. The specification of the aggregate productivity processes will be provided later in Section 5.1.

In each period, after production, the entrepreneur experiences a liquidation shock with probability  $\lambda$ , upon which he loses his idea and needs to liquidate his net worth to return it back to the household.<sup>10</sup> If the liquidation shock happens, the entrepreneur restarts with a draw of a new idea with initial productivity  $\bar{z}$  and an initial net worth  $\chi N_t$  in period t+1, where  $N_t$  is the total (average) net worth of the economy in period t, and  $\chi \in (0,1)$  is a parameter that determines the ratio of the initial net worth of entrepreneurs relative to that of the economy-wide average. Conditional on no liquidation shock, the net worth  $N_{i,t+1}$  of entrepreneur i at time t+1 is determined as

$$N_{i,t+1} = \pi \left( \bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}^d, K_{i,t+1}^{nd} \right) + (1 - \delta_d) q_{d,t+1} K_{i,t+1}^d + (1 - \delta_{nd}) q_{nd,t+1} K_{i,t+1}^{nd} - R_{f,t+1} B_{i,t}.$$

$$(7)$$

The interpretation is that the entrepreneur receives the profit  $\pi\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}^d, K_{i,t+1}^{nd}\right)$  from production. His capital holdings depreciate at rate  $\delta_h$ , and he needs to pay back the debt borrowed from last period plus interest, amounting to  $R_{f,t+1}B_{i,t}$ .

Because of the fact that whenever a liquidity shock occurs, entrepreneurs submit their net worth to the household who chooses consumption collectively for all members, entrepreneurs value their net worth using the same pricing kernel as the household. Let  $V_t^i$  denote the value function of entrepreneur i. It must satisfy the following Bellman equation:

$$V_t^i = \max_{\{K_{i,t+1}^d, K_{i,t+1}^{nd}, N_{i,t+1}, B_{i,t}\}} E_t \left[ M_{t+1} \{ \lambda N_{i,t+1} + (1-\lambda) V_{t+1}^i (N_{i,t+1}) \} \right], \tag{8}$$

subject to the budget constraint (4), the collateral constraint (5), and the law of motion of  $N_{i,t+1}$  given by (7).

We use variables without an i subscript to denote economy-wide aggregate quantities. The aggregate net worth in the entrepreneurial sector satisfies

$$N_{t+1} = (1 - \lambda) \left[ \begin{array}{c} \pi \left( \bar{A}_{t+1}, K_{t+1}^d, K_{t+1}^{nd} \right) + (1 - \delta_d) \, q_{d,t+1} K_{t+1}^d \\ + (1 - \delta_{nd}) \, q_{nd,t+1} K_{t+1}^{nd} - R_{f,t+1} B_t \end{array} \right] + \lambda \chi N_t, \tag{9}$$

where  $\pi\left(\bar{A}_{t+1}, K_{t+1}^d, K_{t+1}^{nd}\right)$  denotes the aggregate profit of all firms.

<sup>&</sup>lt;sup>10</sup>This assumption effectively makes entrepreneurs less patient than the household and prevents them from saving their way out of the financial constraint.

#### 3.3 Production

**Final Output** With  $z_{i,t}$  denoting the idiosyncratic productivity for firm i at time t, output  $y_{i,t}$  of firm i at time t is assumed to be generated through the following production technology:

$$y_{i,t} = \bar{A}_t \left[ z_{i,t}^{1-\nu} \left( K_{i,t}^d + K_{i,t}^{nd} \right)^{\nu} \right]^{\alpha} L_{i,t}^{1-\alpha}$$
(10)

In our formulation,  $\alpha$  is the capital share, and  $\nu$  is the span of control parameter as in Atkeson and Kehoe (2005). Note that durable and non-durable capital are perfect substitutes in production. This assumption is made for tractability.

Firm i's profit at time t,  $\pi\left(\bar{A}_{t}, z_{i,t}, K_{i,t}^{d}, K_{i,t}^{nd}\right)$  is given as

$$\pi \left( \bar{A}_{t}, z_{i,t}, K_{i,t}^{d}, K_{i,t}^{nd} \right) = \max_{L_{i,t}} y_{i,t} - W_{t} L_{i,t},$$

$$= \max_{L_{i,t}} \bar{A}_{t} \left[ z_{i,t}^{1-\nu} \left( K_{i,t}^{d} + K_{i,t}^{nd} \right)^{\nu} \right]^{\alpha} L_{i,t}^{1-\alpha} - W_{t} L_{i,t}, \tag{11}$$

where  $W_t$  is the equilibrium wage rate, and  $L_{i,t}$  is the amount of labor hired by entrepreneur i at time t.

It is convenient to write the profit function explicitly by maximizing out labor in equation (11) and using the labor market clearing condition  $\int L_{i,t} di = 1$  to get

$$L_{i,t} = \frac{z_{i,t}^{1-\nu} \left( K_{i,t}^d + K_{i,t}^{nd} \right)^{\nu}}{\int z_{i,t}^{1-\nu} \left( K_{i,t}^d + K_{i,t}^{nd} \right)^{\nu} di}, \tag{12}$$

so that entrepreneur i's profit function becomes

$$\pi\left(\bar{A}_{t}, z_{i,t}, K_{i,t}^{d}, K_{i,t}^{nd}\right) = \alpha \bar{A}_{t} z_{i,t}^{1-\nu} \left(K_{i,t}^{d} + K_{i,t}^{nd}\right)^{\nu} \left[\int z_{i,t}^{1-\nu} \left(K_{i,t}^{d} + K_{i,t}^{nd}\right)^{\nu} di\right]^{\alpha-1}.$$
 (13)

Given the output of entrepreneur  $i, y_{i,t}$ , from equation (10), the total output of the economy is given as

$$Y_{t} = \int y_{i,t} di,$$

$$= \bar{A}_{t} \left[ \int z_{i,t}^{1-\nu} \left( K_{i,t}^{d} + K_{i,t}^{nd} \right)^{\nu} di \right]^{\alpha}.$$
(14)

**Capital Goods** We assume that capital goods are produced from a constant-return-to-scale and convex adjustment cost function  $G(I, K^d + K^{nd})$ . That is, one unit of the investment good costs  $G(I, K^d + K^{nd})$  units of consumption goods. Therefore, the aggregate

resource constraint is

$$C_t + I_t + G(I_t, K_t^d + K_t^{nd}) = Y_t.$$
 (15)

Without loss of generality, we assume that  $G\left(I_t, K_t^d + K_t^{nd}\right) = g\left(\frac{I_t}{K_t^d + K_t^{nd}}\right) \left(K_t^d + K_t^{nd}\right)$  for a convex function g.

For model tractability, we assume that at the aggregate level, the proportion of two types of capital is fixed, that is,  $\frac{K_t^d}{K_t} = \zeta$ , and  $\frac{K_t^{nd}}{K_t} = 1 - \zeta$ . In order to achieve a fixed proportion, we need to specify  $\phi_t$  and  $1 - \phi_t$  as the fractions of the new investment goods used for typed and type-nd capital, respectively, and  $\phi_t = (\delta_{nd} - \delta_d) \zeta (1 - \zeta) \frac{K_t}{I_t} + \zeta$ . This is another simplification assumption for model tractability. It implies that, at the aggregate level, the ratio of type-d to type-nd capital is always equal to  $\zeta/(1-\zeta)$ , and thus the total capital stock of the economy can be summarized by a single state variable <sup>11</sup>. The aggregate stocks of type-d and type-nd capital satisfy

$$K_{t+1}^d = (1 - \delta_d) K_t^d + \phi_t I_t \tag{16}$$

$$K_{t+1}^{nd} = (1 - \delta_{nd}) K_t^{nd} + (1 - \phi_t) I_t.$$
 (17)

# 4 Equilibrium Asset Pricing

## 4.1 Aggregation

Our economy is one with both aggregate and idiosyncratic productivity shocks. In general, we would have to use the joint distribution of capital and net worth as an infinite-dimensional state variable in order to characterize the equilibrium recursively. In this section, we present a aggregation result as developed in Ai, Li, Li, and Schlag (2019), and show that the aggregate quantities and prices of our model can be characterized without any reference to distributions. Given aggregate quantities and prices, quantities and shadow prices at the individual firm level can be computed using equilibrium conditions.

**Distribution of Idiosyncratic Productivity** In our model, the law of motion of idiosyncratic productivity shocks,  $z_{i,t+1} = z_{i,t}e^{\varepsilon_{i,t+1}}$ , is time invariant, implying that the cross-sectional distribution of the  $z_{i,t}$  will eventually converge to a stationary distribution.<sup>12</sup> At the

<sup>&</sup>lt;sup>11</sup>Without this assumption, we have to keep track of the ratio of two types of capital as an additional aggregate state variable, and we will not not able to achieve the recursion construction of the Markov equilibrium and the aggregation results as shown in Proposition 1.

 $<sup>^{12}</sup>$ In fact, the stationary distribution of  $z_{i,t}$  is a double-sided Pareto distribution. Our model is therefore consistent with the empirical evidence regarding the power law distribution of firm size.

macro level, the heterogeneity of idiosyncratic productivity can be conveniently summarized by a simple statistic:  $Z_t = \int z_{i,t} di$ . It is useful to compute this integral explicitly.

Given the law of motion of  $z_{i,t}$  from equation (6) and the fact that entrepreneurs receive a liquidation shock with probability  $\lambda$ , we have:

$$Z_{t+1} = (1 - \lambda) \int z_{i,t} e^{\varepsilon_{i,t+1}} di + \lambda \bar{z}.$$

The interpretation is that only a fraction  $(1 - \lambda)$  of entrepreneurs will survive until the next period, while the rest will restart with a productivity of  $\bar{z}$ . Note that based on the assumption that  $\varepsilon_{i,t+1}$  is independent of  $z_{i,t}$ , we can integrate out  $\varepsilon_{i,t+1}$  and rewrite the above equation as <sup>13</sup>

$$Z_{t+1} = (1 - \lambda) \int z_{i,t} E\left[e^{\varepsilon_{i,t+1}}\right] di + \lambda \bar{z},$$
  
$$= (1 - \lambda) Z_t e^{\mu_{\varepsilon} + \frac{1}{2}\sigma_{\varepsilon}^2} + \lambda \bar{z},$$
 (18)

where the last equality follows from the fact that  $\varepsilon_{i,t+1}$  is normally distributed. It is straightforward to see that if we choose the normalization  $\bar{z} = \frac{1}{\lambda} \left[ 1 - (1 - \lambda) e^{\mu_{\varepsilon} + \frac{1}{2}\sigma_{\varepsilon}^2} \right]$  and initialize the economy by setting  $Z_0 = 1$ , then  $Z_t = 1$  for all t. This will be the assumption we maintain for the rest of the paper.

**Firm Profits** We assume that  $\varepsilon_{i,t+1}$  is observed at the end of period t when the entrepreneurs plan next period's capital. As we show in Appendix Appendix A, this implies that entrepreneur i will choose  $K_{i,t+t}^d + K_{i,t+1}^{nd}$  to be proportional to  $z_{i,t+1}$  in equilibrium. Additionally, because  $\int z_{i,t+1} di = 1$ , we must have

$$K_{i,t+1}^d + K_{i,t+1}^{nd} = z_{i,t+1} \left( K_{t+1}^d + K_{t+1}^{nd} \right), \tag{19}$$

where  $K_{t+1}^d$  and  $K_{t+1}^{nd}$  are the aggregate quantities of type-d and type-nd capital, respectively.

The assumption that capital is chosen after  $z_{i,t+1}$  is observed rules out capital misallocation and implies that total output does not depend on the joint distribution of idiosyncratic productivity and capital. This is because given idiosyncratic shocks, all entrepreneurs choose the optimal level of capital such that the marginal productivity of capital is the same across all

<sup>&</sup>lt;sup>13</sup>The first line requires us to define the set of firms and the notion of integration in a mathematically careful way. Rather than going to the technical details, we refer the readers to Feldman and Gilles (1985) and Judd (1985). Constantinides and Duffie (1996) use a similar construction in the context of heterogenous consumers. See footnote 5 in Constantinides and Duffie (1996) for a more careful discussion on possible constructions of an appropriate measurable space under which the integration is valid.

entrepreneurs. This fact allows us to write  $Y_t = \bar{A}_t \left( K_t^d + K_t^{nd} \right)^{\alpha \nu} \int z_{i,t} di = \bar{A}_t \left( K_t^d + K_t^{nd} \right)^{\alpha \nu}$ . It also implies that the profit at the firm level is proportional to aggregate productivity, i.e.,

$$\pi\left(\bar{A}_{t}, z_{i,t}, K_{i,t}^{d}, K_{i,t}^{nd}\right) = \alpha \bar{A}_{t} z_{i,t} \left(K_{t}^{d} + K_{t}^{nd}\right)^{\alpha \nu},$$

and the marginal products of capital are equalized across firms for the two types of capital:

$$\frac{\partial}{\partial K_{i,t}^d} \pi \left( \bar{A}_t, z_{i,t}, K_{i,t}^d, K_{i,t}^{nd} \right) = \frac{\partial}{\partial K_{i,t}^{nd}} \pi \left( \bar{A}_t, z_{i,t}, K_{i,t}^d, K_{i,t}^{nd} \right) = \alpha \nu \bar{A}_t \left( K_t^d + K_t^{nd} \right)^{\alpha \nu - 1}. \tag{20}$$

To prove (20), we take derivatives of firm i's output function (10) with respect to  $K_{i,t}^d$  and  $K_{i,t}^{nd}$ , and then impose the optimality conditions (12) and (19).

Intertemporal Optimality Having simplified the profit functions, we can derive the optimality conditions for the entrepreneur's maximization problem (8). Note that given equilibrium prices, the objective function and the constraints are linear in net worth and productivity  $z_{i,t+1}$ . Therefore, the value function  $V_t^i$  must be linear as well. We write  $V_t^i(N_{i,t}, z_{i,t+1}) = \mu_t^i N_{i,t} + \Theta_t^i z_{i,t+1}$ , where  $\mu_t^i$  can be interpreted as the marginal value of net worth for entrepreneur i. Furthermore, let  $\eta_t^i$  be the Lagrangian multiplier associated with the collateral constraint (5). The first order condition with respect to  $B_{i,t}$  implies

$$\mu_t^i = E_t \left[ \widetilde{M}_{t+1}^i \right] R_{t+1}^f + \eta_t^i, \tag{21}$$

where we use the definition

$$\widetilde{M}_{t+1}^{i} \equiv M_{t+1}[(1-\lambda)\,\mu_{t+1}^{i} + \lambda].$$
 (22)

The interpretation is that one unit of net worth allows the entrepreneur to reduce one unit of borrowing, the present value of which is  $E_t\left[\widetilde{M}_{t+1}^i\right]R_{t+1}^f$ , and relaxes the collateral constraint, the benefit of which is measured by  $\eta_t^i$ .

Similarly, the first order condition for  $K_{i,t+1}^d$  is

$$\mu_t^i = E_t \left[ \widetilde{M}_{t+1}^i \frac{\frac{\partial}{\partial K_{i,t+1}^d} \pi \left( \bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}^d, K_{i,t+1}^{nd} \right) + (1 - \delta_d) q_{d,t+1}}{q_{d,t}} \right] + \theta \eta_t^i.$$
 (23)

An additional unit of type-d capital allows the entrepreneur to purchase  $\frac{1}{q_{d,t}}$  units of capital, which pays a profit of  $\frac{\partial \pi}{\partial K^d} \left( \bar{A}_{t+1}, z_{i,t+1}, K^d_{i,t+1}, K^{nd}_{i,t+1} \right)$  over the next period before it depreci-

ates at rate  $\delta_d$ . In addition, a fraction  $\theta$  of type-d capital can be used as collateral to relax the borrowing constraint.

Finally, optimality with respect to the choice of type-nd capital implies

$$\mu_t^i = E_t \left[ \widetilde{M}_{t+1}^i \frac{\frac{\partial}{\partial K_{i,t+1}^{nd}} \pi \left( \bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}^d, K_{i,t+1}^{nd} \right) + (1 - \delta_{nd}) q_{nd,t+1}}{q_{nd,t}} \right] + \theta \eta_t^i. \tag{24}$$

Recursive Construction of the Equilibrium Note that in our model, firms differ in their net worth. First, the net worth depends on the entire history of idiosyncratic productivity shocks, as can be seen from equation (7), since, due to (6),  $z_{i,t+1}$  depends on  $z_{i,t}$ , which in turn depends on  $z_{i,t-1}$  etc. Furthermore, the net worth also depends on the need for capital which relies on the realization of next period's productivity shock. Therefore, in general, the marginal benefit of net worth,  $\mu_t^i$ , and the tightness of the collateral constraint,  $\eta_t^i$ , depend on the individual firm's entire history. Below we show that despite the heterogeneity in net worth and capital holdings across firms, our model allows an equilibrium in which  $\mu_t^i$  and  $\eta_t^i$  are equalized across firms, and aggregate quantities can be determined independently of the distribution of net worth and capital.<sup>14</sup>

The assumptions that type-d and type-nd capital are perfect substitutes in production and that the idiosyncratic shock  $z_{i,t+1}$  is observed before the decisions on  $K^d_{i,t+1}$  and  $K^{nd}_{i,t+1}$  are made imply that the marginal product of both types of capital are equalized within and across firms, as shown in equation (20). As a result, equations (21) to (24) permit solutions where  $\mu^i_t$  and  $\eta^i_t$  are not firm-specific. Intuitively, because the marginal product of capital depends only on the sum of  $K^d_{i,t+1}$  and  $K^{nd}_{i,t+1}$ , but not on the individual summands, entrepreneurs will choose the total amount of capital to equalize its marginal product across firms. This is also because  $z_{i,t+1}$  is observed at the end of period t. Depending on his borrowing need, an entrepreneur can then determine  $K^d_{i,t+1}$  to satisfy the collateral constraint. Because capital can be purchased on a competitive market, entrepreneurs will choose  $K^d_{i,t+1}$  and and  $K^{nd}_{i,t+1}$  to equalize its price to its marginal benefit, which includes the marginal product of capital and the Lagrangian multiplier  $\eta^i_t$ . Because both the prices and the marginal product of capital are equalized across firms, so is the tightness of the collateral constraint.

We formalize the above observation by constructing a recursive equilibrium in two steps. First, we show that the aggregate quantities and prices can be characterized by a set of equilibrium functionals. Second, we further construct individual firm's quantities from the

<sup>&</sup>lt;sup>14</sup>We believe that under our assumptions, this is the only type of equilibrium. However, a rigorous proof is non-trivial and beyond the scope of this paper.

aggregate quantities and prices. We make one final assumption, namely that the aggregate productivity is given by  $\bar{A}_t = A_t (K_{i,t}^d + K_{i,t}^{nd})^{1-\nu\alpha}$ , where  $\{A_t\}_{t=0}^{\infty}$  is an exogenous Markov productivity process. On the one hand, this assumption follows Frankel (1962) and Romer (1986) and is a parsimonious way to generate endogenous growth. On the other hand, combined with recursive preferences, this assumption increases the volatility of the pricing kernel, as in the stream of long-run risk model (see, e.g., Bansal and Yaron (2004) and Kung and Schmid (2015)). From a technical point of view, thanks to this assumption, equilibrium quantities are homogenous of degree one in the total capital stock,  $K^d + K^{nd}$ , and equilibrium prices do not depend on  $K^d + K^{nd}$ . It is therefore convenient to work with normalized quantities.

Let lower case variables denote aggregate quantities normalized by the current capital stock, so that, for instance,  $n_t$  denotes aggregate net worth  $N_t$  normalized by the total capital stock  $K^d + K^{nd}$ . The equilibrium objects are consumption, c(A, n), investment, i(A, n), the marginal value of net worth,  $\mu(A, n)$ , the Lagrangian multiplier on the collateral constraint,  $\eta(A, n)$ , the price of type-d capital,  $q_d(A, n)$ , the price of type-nd capital,  $q_{nd}(A, n)$ , and the risk-free interest rate,  $R_f(A, n)$  as functions of the state variables A and n.

To introduce the recursive formulation, we denote a generic variable in period t as X and in period t + 1 as X'. Given the above equilibrium functionals, we can define

$$\Gamma\left(A,n\right) \equiv \frac{K'^{d} + K'^{nd}}{K^{d} + K^{nd}} = \left(1 - \delta_{nd}\right) + \left(\delta_{nd} - \delta_{d}\right)\zeta + i\left(A,n\right)$$

as the growth rate of the capital stock and construct the law of motion of the endogenous state variable n from equation (9):<sup>15</sup>

$$n' = (1 - \lambda) \begin{bmatrix} \alpha \nu A' + \zeta (1 - \delta_d) q_d (A', n') + (1 - \zeta) (1 - \delta_{nd}) q_{nd} (A', n') \\ -\theta [\zeta q_d (A, n) + (1 - \zeta) q_{nd} (A, n)] R_f (A, n) \end{bmatrix} + \lambda \chi \frac{n}{\Gamma (A, n)}.$$
 (25)

With the law of motion of the state variables, we can construct the normalized utility of the household as the fixed point of

$$u(A,n) = \left\{ (1-\beta)c(A,n)^{1-\frac{1}{\psi}} + \beta\Gamma(A,n)^{1-\frac{1}{\psi}} \left( E[u(A',n')^{1-\gamma}] \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}}.$$

The make use of the property that the ratio of  $K_t^d$  over  $K_t^{nd}$  is always equal to  $\zeta/(1-\zeta)$ , as implied by the law of motion of the capital stock in equation (17).

The stochastic discount factors can then be written as

$$M' = \beta \left[ \frac{c(A', n') \Gamma(A, n)}{c(A, n)} \right]^{-\frac{1}{\psi}} \left[ \frac{u(A', n')}{E\left[u(A', n')^{1-\gamma}\right]^{\frac{1}{1-\gamma}}} \right]^{\frac{1}{\psi} - \gamma}$$
(26)

$$\widetilde{M}' = M'[(1-\lambda)\mu(A',n') + \lambda]. \tag{27}$$

Formally, an equilibrium in our model consists of a set of aggregate quantities,  $\{C_t, B_t, \Pi_t, K_t^d, K_t^{nd}, I_t, N_t\}$ , individual entrepreneur choices,  $\{K_{i,t}^d, K_{i,t}^{nd}, L_{i,t}, B_{i,t}, N_{i,t}\}$ , and prices  $\{M_t, \widetilde{M}_t, W_t, q_{d,t}, q_{nd,t}, \mu_t, \eta_t, R_{f,t}\}$  such that, given prices, quantities satisfy the household's and the entrepreneurs' optimality conditions, the market clearing conditions, and the relevant resource constraints. Below, we present a procedure to construct a Markov equilibrium where all prices and quantities are functions of the state variables (A, n). For simplicity, we assume that the initial idiosyncratic productivity across all firms satisfies  $\int z_{i,1} di = 1$ , the initial aggregate net worth is  $N_0$ , aggregate capital holdings start with  $\frac{K_1^d}{K_1^{nd}} = \frac{\zeta}{1-\zeta}$ , and firm's initial net worth satisfies  $n_{i,0} = z_{i,1} N_0$  for all i.

Again we use, x and X to denote a generic normalized and non-normalized quantity, respectively. For example, c denotes normalized aggregate consumption, while C is the original value.

#### **Proposition 1.** (Markov Equilibrium)

Suppose there exists a set of equilibrium functionals  $\{c(A,n), i(A,n), \mu(A,n), \eta(A,n), q_d(A,n), q_d(A,n), R_f(A,n), \phi(A,n)\}$  satisfying the following set of functional equations:

$$E[M'|A]R_f(A,n) = 1,$$
 (28)

$$\mu(A,n) = E\left[\widetilde{M}'\middle|A\right] R_f(A,n) + \eta(A,n), \qquad (29)$$

$$\mu(A,n) = E\left[\widetilde{M}'\frac{\alpha\nu A' + (1-\delta_d)q_d(A',n')}{q_d(A,n)}\middle|A\right] + \theta\eta(A,n),$$
(30)

$$\mu(A,n) = E\left[\widetilde{M}'\frac{\alpha\nu A' + (1-\delta_{nd}) q_{nd}(A',n')}{q_{nd}(A,n)} \middle| A\right] + \theta\eta(A,n),$$
(31)

$$\frac{n}{\Gamma(A,n)} = (1-\theta)\zeta q_d(A,n) + (1-\theta)(1-\zeta)q_{nd}(A,n), \qquad (32)$$

$$G'(i(A,n)) = \phi(A,n) q_d(A,n) + (1 - \phi(A,n)) q_{nd}(A,n),$$
(33)

$$c(A, n) + i(A, n) + g(i(A, n)) = A,$$
 (34)

$$\phi(A,n) = \frac{\left(\delta_{nd} - \delta_d\right)\left(1 - \zeta\right)\zeta}{i(A,n)} + \zeta \tag{35}$$

where the law of motion of n is given by (A4), and the stochastic discount factors M' and  $\widetilde{M'}$  are defined in (A5) and (A6). Then the equilibrium prices and quantities can be constructed as follows and they constitute a Markov equilibrium:

1. Given the sequence of exogenous shocks  $\{A_t\}$ , the sequence of  $n_t$  can be constructed using the law of motion in  $(A_t)$ , firm's value function is of the form  $V_t^i(N_{i,t}, z_{i,t+1}) = \mu(A_t, n_t) N_{i,t} + \theta(A_t, n_t) \left(K_t^d + K_t^{nd}\right) z_{i,t+1}$ , the normalized policy functions are constructed as:

$$x_t = x(A_t, n_t), \text{ for } x = c, i, \mu, \eta, q_d, q_{nd}, R_f, \phi,$$

and are jointly determined by Equations (28)-(35). The normalized value function  $\theta(A_t, n_t)$  is given in Equation (A16) in Section Appendix A in the Appendix.

2. Given the sequence of normalized quantities, aggregate quantities are constructed as:

$$K_{t+1}^{d} = K_{t}^{d} \left[ 1 - \delta_{d} + \phi_{t} i_{t} \right], \quad K_{t+1}^{nd} = K_{t}^{nd} \left[ 1 - \delta_{nd} + (1 - \phi_{t}) i_{t} \right]$$

$$X_{t} = x_{t} \left[ K_{t}^{d} + K_{t}^{nd} \right]$$

for x = c, i, b, n, X = C, I, B, N, and all t.

3. Given the aggregate quantities, the individual entrepreneurs' net worth follows from (7). Given the sequences  $\{N_{i,t}\}$ , the quantities  $B_{i,t}$ ,  $K_{i,t}^d$  and  $K_{i,t}^{nd}$  are jointly determined by equations (4), (5), and (19). Finally,  $L_{i,t} = z_{i,t}$  for all i,t.

The above proposition implies that we can solve for aggregate quantities first, and then use the firm-level budget constraint and the law of motion of idiosyncratic productivity in to construct the cross-section of net worth and capital holdings. Note that our construction of the equilibrium allows  $\eta(A, n) = 0$  for some values of (A, n). That is, our general setup allows occasionally binding constraints. Numerically, we use a local approximation method to solve the model by assuming the constraint is always binding.

In our model, firm value function,  $V(N_{i,t}, z_{i,t+1}) = \mu(A_t, n_t) N_{i,t} + \theta(A_t, n_t) \left(K_t^d + K_t^{nd}\right) z_{i,t+1}$  has two components:  $\mu(A_t, n_t) N_{i,t}$  is the present value of net worth and  $\theta(A_t, n_t) \left(K_t^d + K_t^{nd}\right) z_{i,t+1}$  is the present value of profit. In the special case of constant returns to scale,  $\theta(A_t, n_t) = 0$  because firms do not make any profit. The general expression for  $\theta(A, n)$  is provided in Appendix A. By the above proposition, other equilibrium quantities are jointly determined by conditions (28)-(35) independent of the functional form of  $\theta(A, n)$ . This is because  $z_{i,t+1}$  is exogenously given and does not affect the determination of equilibrium

optimality conditions.

The above conditions have intuitive interpretations. Equation (28) is the household's intertemporal Euler equation with respect to the choice of the risk-free asset. Equation (29) is the firm's optimality condition for the choice of debt. Equations (30) and (31) are the firm's first-order conditions with respect to the choice of type-d and type-nd capital. Equation (32) is the binding budget constraint of firms, Equation (33) is the optimality condition for capital goods production, Equation (34) is the aggregate resource constraint, and Equation (35) gives the allocation of new investment into two types of capital to ensure a fixed proportion of type-d and type-nd capital at the aggregate. Proposition 1 implies that conditions (28)-(35) are not only necessary but also sufficient for the construction of the equilibrium quantities.

In our model, because type-d capital can perfectly substitute for type-nd capital in production and both types of capital are freely traded on the market, the marginal product of capital must be equalized within and across firms. The trading of capital therefore equalizes the Lagrangian multiplier of the financial constraints across firms. This is the key feature of our model that allows us to construct a Markov equilibrium without having to include the distribution of capital as a state variable.  $^{16}$ 

## 4.2 Trade-off between User Cost and Down Payment

As mentioned in Proposition 1, the aggregate quantities and prices do not depend on the joint distribution of individual entrepreneur level capital and net worth. In this section we define the user costs of type-d (type-nd) capital in the presence of collateral constraint and aggregate risks by extending the definition in Jorgenson (1963). The optimal decision to choose type-d versus type-nd capital is achieved when the user costs of two types of capital are equalized. The definitions in this section clarify a novel risk premium channel of type-d (type-nd) capital, which has not been emphasized in prior literature.

The user cost of capital,  $\tau_{h,t}$ ,  $h \in \{d, nd\}$ , is determined as:

<sup>&</sup>lt;sup>16</sup>Because of these simplifying assumptions, our model is silent on why some firms are constrained and others are not.

$$\begin{split} \tau_{h,t} &= q_{h,t} \left( 1 - \theta \right) - E_t \left[ \frac{\widetilde{M}_{t+1}}{\mu_t} \left\{ q_{h,t+1} \left( 1 - \delta_h \right) - R_{f,t+1} \theta q_{h,t} \right\} \right] \\ &= \vartheta_{h,t} - \left( 1 - \delta_h \right) \left[ \frac{1}{R_{I,t+1}} E_t \left[ q_{h,t+1} \right] + Cov_t \left( \frac{\widetilde{M}_{t+1}}{\mu_t}, q_{h,t+1} \right) \right] + \frac{R_{f,t+1}}{R_{I,t+1}} \theta q_{h,t} \\ &= \vartheta_{h,t} + \left( 1 - \delta_h \right) Cov_t \left( \frac{\widetilde{M}_{t+1}}{\mu_t}, q_{h,t+1} \right) - \frac{1}{R_{I,t+1}} E_t \left[ q_{h,t+1} \left( 1 - \delta_h \right) - R_{f,t+1} \theta q_{h,t} \right] \end{split}$$

The interpretation is that the user cost of type-d (type-nd) capital is equal to the minimum down payment per unit of capital paid upfront,  $q_{h,t}(1-\theta)$ , minus the present value of the fractional resale value next period that cannot be pledged, based on the first equality.

We further provide intuition about the trade-off underlying the type-d versus type-nd decisions by comparing the user costs of type-d (type-nd) capital. Let us first define two important wedges to reveal the relationship. First, we denote a shadow interest rate for the borrowing and lending among entrepreneurs  $R_{I,t}$ , and it is determined by:

$$1 = E_t \left(\frac{\widetilde{M}_{t+1}}{\mu_t}\right) R_{I,t+1}. \tag{36}$$

Based on equation (21) and the above definition (36), we can derive that there is a wedge,  $\Delta_{f,t}$ , between two interest rates,

$$\Delta_{f,t} = R_{I,t} - R_{f,t} = \frac{\eta_t}{\mu_t} R_{I,t}.$$

When the collateral constraint is binding  $(\eta_t > 0)$ , this wedge becomes strictly positive. It reflects a premium that entrepreneurs has to pay for the loans among themselves, when cheaper household loans become unaccessible due to a binding collateral constraint.

Second, we denote an risk premium wedge,  $\Delta_{rp,t}$ , as the difference in the risk premium evaluated by entrepreneurs' stochastic discount factors for type-d versus type-nd capital, as below:

$$\Delta_{rp,t} = -Cov_t\left(\frac{\widetilde{M}_{t+1}}{\mu_t}, q_{d,t+1}\right) + Cov_t\left(\frac{\widetilde{M}_{t+1}}{\mu_t}, q_{nd,t+1}\right).$$

With the help of the above two wedges, we can decompose the difference in user costs of type-d capital versus type-nd capital as below.

$$\tau_{d,t} - \tau_{nd,t} = (\vartheta_{d,t} - \vartheta_{nd,t}) + \Delta_{rp,t} - \frac{1}{R_{f,t+1} + \Delta_{f,t+1}} \begin{bmatrix} E_t (q_{d,t+1} (1 - \delta_d) - \theta R_{f,t} q_{d,t}) \\ -E_t (q_{nd,t+1} (1 - \delta_{nd}) - \theta R_{f,t} q_{nd,t}) \end{bmatrix}$$

The left hand side of the above equation reflects the difference in user cost with respect to type-d and type-nd capital. The first two terms on the right hand side reflect the cost of using durable capital. From the perspective of a financially constrained firm, it is costly for him to buy durable capital for two reasons. First, according to the first component in the above equation, durable capital is costly because it requires more down payment; second, according to the second component, durable capital requires higher risk premium. The intuition is the following: due to the fact that the collateral constraint becomes tighter in recessions, the price of type-d capital is more procyclical that that of type-nd capital. Therefore,  $q_{d,t+1}$  is more negatively covaried with with entrepreneurs' augmented stochastic discount factor. Therefore,  $\Delta_{rp,t} > 0$ . This risk premium wedge implies additional user cost of acquiring more durable cost, by paying an additional risk premium, as compared with using less durable capital. The first term has been emphasized by Rampini (2019), while the second risk premium component is a key novel channel that we emphasize in the paper.

The last term,  $\frac{1}{R_{f,t+1}+\Delta_{f,t+1}}\begin{bmatrix} E_t\left(q_{d,t+1}\left(1-\delta_d\right)-\theta R_{f,t}q_{d,t}\right) \\ -E_t\left(q_{nd,t+1}\left(1-\delta_{nd}\right)-\theta R_{f,t}q_{nd,t}\right) \end{bmatrix}$ , denotes the difference in the present value of capital resale value next period that cannot be pledged, subject to depreciation. This term is positive, and reflects the benefit of acquiring durable capital. Because the durable capital has lower depreciation rate, therefore, its next period resale value is larger.

As the financial constraint becomes tighter, the cost of acquiring durable capital, i.e. more expensive down payment and a higher risk premium, will become larger, while the benefit (last term) will become less important due to an increasing in interest rate wedge,  $\Delta_f$ . In the extreme case, in which the firm is infinitely constraint, that is,  $\Delta_f$  goes to infinity, the last term disappears, then the asset durability decision purely depends on a comparison of down payment and risk premium.

Taken together, the key contribution in our paper is to highlight an additional risk premium channel by building a dynamic choice of asset durability into a general equilibrium model with financial frictions and aggregate risks.

Consider a special case which can flesh out our contribution. If there is no adjustment

cost, then  $q_h$  is constant, which implies that

$$\tau_{d,t} - \tau_{nd,t} = (\vartheta_{d,t} - \vartheta_{nd,t})$$

$$-\frac{1}{R_{f,t+1} + \Delta_{f,t+1}} \begin{bmatrix} q_d (1 - \delta_d - \theta R_{f,t}) \\ -q_{nd} (1 - \delta_{nd} - \theta R_{f,t}) \end{bmatrix}$$

Importantly, in this case, capital prices do not fluctuate, thus the risk premium wedge  $\Delta_{rp,t}$  disappears. The asset durability trade-off goes back to Rampini (2019). The key contribution of our paper is to point out an additional risk premium channel through a general equilibrium model with financial frictions and aggregate risks, and further empirically quantify it through the lens of cross-section of equity returns.

## 4.3 Asset Pricing Implications

In this section we study the asset pricing implications of the model both at the aggregate and firm level.

Asset Durability Spread at the Aggregate Level Our model allows for two types of capital, where the depreciation rate of type-d capital is lower than that of type-nd capital. We define the return on the type-d capital and type-nd capital, respectively, and discuss their different risk profiles. Note that one unit of type h capital costs  $q_{h,t}$  in period t and it pays off  $\Pi_{i,t+1} + (1 - \delta_h) q_{h,t+1}$  in the next period, for  $h \in \{d, nd\}$ . Therefore, the un-levered returns on the claims to type-d (type-nd) capital are given by:

$$R_{h,t+1} = \frac{\alpha \nu A_{t+1} + (1 - \delta_h) q_{h,t+1}}{q_{h,t}} \quad (h = d, nd). \tag{37}$$

In analogy to its un-levered return, the levered return of type-d (type-nd) capital denotes as

$$R_{h,t+1}^{Lev} = \frac{\alpha \nu A_{t+1} + (1 - \delta_h) q_{h,t+1} - R_{f,t+1} \theta (1 - \delta_h) q_{h,t}}{q_{h,t} (1 - \theta)},$$

$$= \frac{1}{1 - \theta} (R_{h,t+1} - R_{f,t+1}) + R_{f,t+1}. \tag{38}$$

The denominator  $q_{h,t}(1-\theta)$  denotes the amount of internal net worth required to buy one unit of capital, and it can be interpreted as the minimum down payment per unit of capital. The numerator  $\alpha \nu A_{t+1} + (1-\delta_h) q_{h,t+1} - R_{f,t+1} \theta q_{h,t}$  is tomorrow's payoff per unit of capital, after subtracting the debt repayment. Therefore,  $R_{h,t+1}^{Lev}$  is a levered return. Clearly, the levered return implied leverage ratio is  $\frac{1}{1-\theta}$ .

Undoubtedly, risk premia are determined by the covariance of the payoffs with respect to the stochastic discount factor. Given that the components representing the marginal products of capital in the payoff are identical for the two types of capital, the key to understand the asset durability premium depends on the fact that the depreciated resale value of type-d capital is subject to higher aggregate exposures than that of type-nd capital. In the other words, the asset durability premium, as shown later, is driven by the difference in cyclical properties of the price with respect to two types of capital,  $q_{h,t+1}$ .

Combine the two Euler equations, (21) and (23), and eliminate  $\eta_t$ , we have

$$E_t\left[\widetilde{M}_{t+1}R_{d,t+1}^{Lev}\right] = \mu_t,$$

and the rearrangement in the equation (24) gives

$$E_t \left[ \widetilde{M}_{t+1} R_{nd,t+1}^{Lev} \right] = \mu_t.$$

Therefore, the expected return spread is equal to

$$E_{t}\left(R_{d,t+1}^{Lev}-R_{nd,t+1}^{Lev}\right) = -\frac{1}{E_{t}\left(\widetilde{M}_{t+1}\right)}\left(Cov_{t}\left[\widetilde{M}_{t+1},R_{d,t+1}^{Lev}\right] - Cov_{t}\left[\widetilde{M}_{t+1},R_{nd,t+1}^{Lev}\right]\right). \tag{39}$$

As shown in equation (39), risk premia are determined by the covariance of the stochastic discount factor and the payoff with respect to each type of capital. Apparently, we notice that the main driving force of return variations comes from the resale price  $(1 - \delta_d) q_{d,t+1}$  rather than from the marginal product of capital component. The resale price of type-d capital, as exhibiting a higher cyclicality, is more covaried with the stochastic discount factor. Hence,  $R_{d,t+1}^{Lev}$  is more more risky than its counterparty  $R_{nd,t+1}^{Lev}$ . Overall, the right hand side of equation (39) is positive, that is, type-d capital earns a higher expected return than type-nd capital. Up to now, our model in this subsection shows a positive asset durability premium at the aggregate level.

Asset Durability Spread at the Firm Level In our model, equity claims to firms can be freely traded among entrepreneurs. In our calibrated model,  $\nu$  is close to one, and the profit component is much smaller than that of the net worth component. Recall that  $\theta_t = 0$  when  $\nu = 1$  in Equation (A16) of Appendix Appendix A. We therefore define the equity returnreturn on an entrepreneur's net worth approximately to be  $\frac{N_{i,t+1}}{N_{i,t}}$ . Using (4) and (7), we can write this return as

$$R_{i,t+1} = \frac{\alpha \nu A_{t+1} \left( K_{i,t+1}^d + K_{i,t+1}^{nd} \right) + (1 - \delta_d) q_{d,t+1} K_{i,t+1}^d + (1 - \delta_{nd}) q_{nd,t+1} K_{i,t+1}^{nd} - R_{f,t+1} B_{i,t}}{N_{i,t}}$$

$$= \frac{(1 - \theta) q_{d,t} K_{i,t+1}^d}{N_{i,t}} R_{d,t+1}^{Lev} + \frac{(1 - \theta) q_{nd,t} K_{i,t+1}^{nd}}{N_{i,t}} R_{nd,t+1}^{Lev}.$$

The above expression has an intuitive interpretation: the firm's equity return is a weighted average of the levered return on type-d capital,  $R_{d,t+1}^{Lev}$ , and the return on type-nd capital,  $R_{nd,t+1}^{Lev}$ . The weights  $\frac{(1-\theta)q_{d,t}K_{i,t+1}^d}{N_{i,t}}$  and  $\frac{(1-\theta)q_{nd,t}K_{i,t+1}^n}{N_{i,t}}$  are the fraction of the down payment in the entrepreneur i's net worth. Moreover, these weights are sum up to one, as restricted by the budget constraint and the binding collateral constraint.

In our model,  $R_{d,t+1}^{Lev}$  and  $R_{nd,t+1}^{Lev}$  are common across all firms. As a result, expected returns differ across firms only because of the composition of expenditure on type-d versus the type-nd capital. Such the composition of expenditure is equivalently summarized by the measure of asset durability. As shown the next section, this parallel between our model and our empirical results allows our model to match well the quantitative features of the asset durability spread in the data.

# 5 Quantitative Model Predictions

In this section, we calibrate our model at the annual frequency and evaluate its ability to replicate key moments of both macroeconomic quantities and asset prices at the aggregate level. More importantly, we investigate its performance in terms of quantitatively accounting for key features of firm characteristics and producing an asset durability premium in the cross-section. For macroeconomic quantities, we focus on a long sample of U.S. annual data from 1930 to 2017. All macroeconomic variables are real and per capita. Consumption, output and physical investment data are from the Bureau of Economic Analysis (BEA). For the purpose of cross-sectional analyses we make use of several data sources at the micro-level, which is summarized in Appendix B.

## 5.1 Specification of Aggregate Shocks

In this section, we formalize the specification of the exogenous aggregate shocks in this economy. First, log aggregate productivity  $a \equiv \log(A)$  follows

$$a_t = a_{ss} \left( 1 - \rho_A \right) + \rho_A a_{t-1} + \sigma_A \varepsilon_{A,t}, \tag{40}$$

where  $a_{ss}$  denotes the steady-state value of a. Second, as in Ai, Li, and Yang (2018), we also introduce a aggregate shock to entrepreneurs' liquidation probability  $\lambda$ . We interpret it as a shock originating directly from the financial sector, in a spirit similar to Jermann and Quadrini (2012). We introduce this extra source of shocks mainly to improves the quantitative performance of the model. As in all standard real business cycle models, with just an aggregate productivity shock, it is hard to generate large enough variations in capital prices and the entrepreneurs' net worth so that they become consistent with the data.

Importantly, however, our general model intuition that non-durable capital is less risky than durable capital holds for both productivity and financial shocks. The shock to the entrepreneurs' liquidation probability directly affects the entrepreneurs' discount rate, as can be seen from (A6), and thus allows to generate stronger asset pricing implications.<sup>17</sup>

Note that technically  $\lambda \in (0,1)$ . For parsimony, we set

$$\lambda_t = \frac{\exp(x_t)}{\exp(x_t) + \exp(-x_t)},$$

and  $x_t$  itself follows and autocorrelated process:

$$x_t = x_{ss}(1 - \rho_x) + \rho_x x_{t-1} + \sigma_x \varepsilon_{x,t}.$$

We assume the innovations:

$$\left[\begin{array}{c} \varepsilon_{A,t+1} \\ \varepsilon_{x,t+1} \end{array}\right] \sim Normal\left(\left[\begin{array}{c} 0 \\ 0 \end{array}\right], \left[\begin{array}{cc} 1 & \rho_{A,x} \\ \rho_{A,x} & 1 \end{array}\right]\right),$$

in which the parameter  $\rho_{A,x}$  captures the correlation between these two shocks. In the benchmark calibration, we assume the correlation coefficient  $\rho_{A,x} = -1$ . First, a negative correlation indicates that a negative productivity shock is associated with a positive discount rate shock. This assumption is necessary to quantitatively generate a positive correlation

<sup>&</sup>lt;sup>17</sup>Macro models with financial frictions, for instance, Gertler and Kiyotaki (2010) and Elenev et al. (2018), use a similar device for the same reason.

between consumption and investment growth that is consistent with the data. If only the financial shock innovation,  $\varepsilon_{x,t+1}$ , is open, such an innovation will not affect the contemporaneous output. The resource constraint in equation (15) implies a contractually negative correlation between consumption and investment growth. Second, the assumption of a perfectly negative correlation is for parsimony and enables the economy to effectively narrow down to one shock.

#### 5.2 Calibration

We calibrate our model at the quarterly frequency. Table 4 reports the list of parameters and the corresponding macroeconomic moments in our calibration procedure. We group our parameters into four blocks. In the first block, we list the parameters which can be determined by the previous literature. In particular, we set the relative risk aversion  $\gamma$  to be 10 and the intertemporal elasticity of substitution  $\psi$  to be 2. These are parameter values in line with the long-run risks literature, e.g., Bansal and Yaron (2004). The capital share parameter,  $\alpha$ , is set to be 0.30, close to the number used in the standard RBC literature, e.g., Kydland and Prescott (1982) The span of control parameter  $\nu$  is set to be 0.90, consistent with Atkeson and Kehoe (2005).

### [Place Table 4 about here]

The parameters in the second block are determined by matching a set of first moments of quantities and prices to their empirical counterparts. We set the average economy-wide productivity growth rate  $E(A_{ss})$  to match a mean growth rate of U.S. economy of 2% per year. The time discount factor  $\beta$  is set to match the average real risk free rate of 1% per year. The depreciation rate for the durable (non-durable) capital is set to match a 1(3)% annual capital depreciation rate in the data. The average entrepreneur exit probability  $E(\lambda)$  is calibrated to be 0.025, roughly matching to an average Compustat age of 10 years for financially constrained firms. We calibrate the remaining two parameters related to financial frictions, namely, the collateralizability parameter,  $\theta$ , and the transfer to entering entrepreneurs,  $\chi$ , by jointly matching two moments. The average leverage ratio is 0.31 and the average consumption to investment ratio E(C/I) is 4. The targeted leverage ratio is broadly in line with the median of U.S. non-financial firms in Compustat.

The parameters in the third block are not directly related to the first moment of the economy, but they are determined by the second moments in the data. The persistence parameter  $\rho_A$  and  $\rho_x$  are calibrated to be the at 0.994 and 0.98, respectively, roughly matching

the autocorrelation of consumption and output growth. The standard deviation of the  $\lambda$  shock,  $\sigma_x$ , and that of the productivity shock,  $\sigma_A$ , are jointly calibrated to match the volatility of consumption growth and the correlation between consumption and investment growth. The elasticity parameter of the investment adjustment cost functions,  $\zeta$ , is set to allow our model to achieve a sufficiently high volatility of investment, in line with the data.

The last block contains the parameters related to idiosyncratic productivity shocks. We calibrate them to match the mean and standard deviation of the idiosyncratic productivity growth of financially constrained firms in the U.S. Compustat database.

#### 5.3 Numerical Solution and Simulation

As we shown in Section 2.1, financially constrained firm use less durable assets, and the asset durability premium is mainly driven by financially constrained firms. Therefore, we intensionally calibrate our model parameters and thus render the collateral constraint to be binding at the steady state. As a result, our model implications mainly focus on financially constrained firms. This feature of the calibration also simplifies our computation. To be specific, we follow the prior macroeconomic literature, for instance, Gertler and Kiyotaki (2010), to assume the constraint is binding over the narrow region around the steady state. Thus, the local approximation solution method is a good approximation. We solve the model using a second-order local approximation around the risky steady state, and the solution is computed by using the Dynare++ package.

We report the model simulated moments in the aggregate and the cross-section, and compare them to the data. We simulate the model at the annual frequency. Each simulation has a length of 60 years. We drop the first 10 years of each simulation to avoid dependence on initial values and repeat the process 100 times. At the cross-sectional level, each simulation contains 5,000 firms.

## 5.4 Aggregate Moments

In this section, we focus on the quantitative performance of the model at the aggregate level and document the success of our model to match a wide set of conventional moments in macroeconomic quantities and asset prices. More importantly, our model delivers a sizable asset durability spread at the aggregate level.

Table 5 reports the key moments of macroeconomic quantities (top panel) and those of asset returns (bottom panel), respectively, and compares them to their counterparts in the data

where available. The top panel shows that the model simulated data are broadly consistent with the basic features of the aggregate macro-economy in terms of volatilities, correlations, and persistence of output, consumption, and investment. In sum, our model maintains the success of neoclassical growth models in accounting for the dynamics of macroeconomic quantities.

#### [Place Table 5 about here]

Focusing on the asset pricing moments (bottom panel), we make two observations. First, our model is reasonably successful in generating asset pricing moments at the aggregate level. In particular, it replicates a low and smooth risk free rate, with a mean of 1.15% and a volatility of 0.80%. The equity premium in this economy is 6.82%, broadly consistent with the empirical target of 5.71% in the data. Second, our model is also able to generate the levered return on durable capital,  $E[R_{nd}^{Lev} - R_f]$ , at 5.50% and levered return on non-durable capital,  $E[R_{nd}^{Lev} - R_f]$ , at 1.50%. More importantly, our model succeeds to generate a sizable average return spread between return on two types of capital.

## 5.5 Impulse Response Functions

The asset pricing implications of our model are best illustrated with impulse response functions.

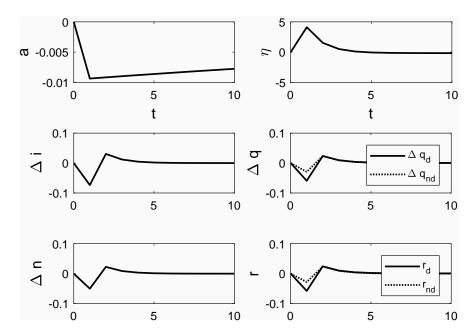
In Figure 1, we plot the percentage deviations of quantities and prices from the steady state in response to a one-standard deviation productivity shock, i.e. the shock to a. The used parameters are corresponding to Table 4. The only one exception in the above figure is that the financial shock,  $\varepsilon_x$ , is orthogonal to the productivity shock,  $\varepsilon_A$ . In the other words,  $\rho_{A,x} = 0$ . Our motivation to shut down the correlation is to highlight the separate effect from a purely productivity shock and we also want to point out the major departure of the model with an orthogonal productivity shock from the benchmark model with correlated shocks.

Three observations are summarized as follows. First, a positive shock to a (top panel in the left column) works as a positive discount rate shock to entrepreneurs, and the shock leads to a tightening of the collateral constraint as reflected by a spike in the Lagrangian multiplier,  $\eta$  (top panel in the right column).

Second, a tightening of the collateral constraints translate into a lower investment (second panel in the left column). Upon a negative productivity shock, not only entrepreneur net worth drops sharply (third panel in the left column), but also the price of type-d capital falls sharply (second panel in the right column). However, the price of type-nd capital falls

Figure 1: Impulse Responses to the Productivity Shock

This figure plots the log-deviations from the steady state for quantities and prices with respect to a one-standard deviation shock to the a. One period is a year. All parameters are calibrated as in Table 4.



much smaller, in contrast to the price of type-d capital. This observation suggests that the price type-d presents higher fluctuations to aggregate shocks, which is consistent with our key model implications.

Lastly and most importantly, the different risk profiles are reflected in different responses of the levered return on type-d capital,  $r_d$ , and that on type-nd capital,  $r_{nd}$ . The return of type-d capital responds much more to negative productivity shocks than that of type-nd capital (bottom panel in the right column). This is because, in recessions, when firms are collectively more constrained, they will prefer "cheap" type-nd capital, making the price of type-d capital declines more significantly as shown in the second panel in the right column. In summary, the levered return on type-d capital,  $r_d^{Lev}$  responds much stronger than the levered return on type-d capital,  $r_{nd}$ , suggesting that durable capital is indeed more risky than non-durable capital in our model, and creates a large expected return spread at the aggregate level.

## 5.6 Asset Durability Spread

We now turn to the implications of our model on the cross-section of asset durability-sorted portfolios. We simulate firms from the model, measure the durability of firm assets, and

conduct the same asset durability-based portfolio-sorting procedure as in the data. In Table 6, we report the average returns of the sorted portfolios along with several other characteristics from the data and those from the simulated model.

### [Place Table 6 about here]

As in the data, firms with high asset durability have a significantly higher average return than those with low asset durability in our model. Quantitatively, our model produces a sizable asset durability spread of around 3.63%, accounting for more than 50% of the spread in the data.

Table 6 also reports several other characteristics of the asset durability-sorted portfolios that are informative about the economic mechanism we emphasize in our model. First, not surprisingly, the asset durability measure is monotonically increasing for asset durability-sorted portfolios. In fact, asset durability in our model is similar in magnitude to that in the data.

Second, as in the data, leverage is increasing in asset durability. This implication of our model is consistent with the data and the broader corporate finance literature. The dispersion in leverage in our model is somewhat higher than in the data. This finding is not surprising, as in our model, each unit of capital can support  $\theta(1 - \delta_h)$  units of borrowing. Each unit of durable capital can support more debt with a lower depreciation rate.

Third, as in the data, high asset durability firms also tend to have higher return on equity (ROE). In our model, other things being equal, firms that experienced a history of positive productivity shocks have a higher financial need and optimally chose to obtain higher asset durability. In the model, a history of higher productivity shocks is also associated with higher ROE. As we show in Table 6, this feature of our model is also consistent with the pattern in the data.

## 6 Empirical Analysis

In this section, we first provide direct empirical evidence for the positive relation between asset durability and capital price cyclicality. Differential fluctuations in capital price translate into the cross-section of stock returns. Next, we perform a battery of asset pricing factor tests to show that such a positive relation is largely unaffected by known return factors for other systematic risks, especially controlling for the collaterizability premium. We then investigate the joint link between durability and other firm-level characteristics on one hand and future

stock returns in the cross-section on the other using Fama and MacBeth (1973) regressions as a valid cross-check for the positive relation between asset durability and stock returns.

## 6.1 Aggregate Shocks and Price Dynamics

Financial conditions among firms exacerbate during economic downturns, given that financial constraints are more binding. Meanwhile, more financially constrained firms tend to acquire "cheaper" less durable assets with lower requirements for down payments. Hence, the price of these preferable assets appears less procyclical and is therefore less risky than that of durable assets. Our model predicts that less durable assets, in contrast to durable assets, are less risky to provide insurance against aggregate shocks. In this subsection, we show the direct evidence to support the prediction that the capital price of more durable asset presents higher sensitivity to macroeconomic shocks as compared with that of less durable capital.

We proceed as follows. First, we measure the log price changes  $(\Delta q_{h,t})$  in each assets according to NIPA tables from the Bureau of Economic Analysis (BEA)<sup>18</sup>. Aggregate macroeconomic shocks  $(\Delta y_t)$  are proxied by the log difference of GDP.<sup>19</sup> In the second step, we estimate exposures by regressing asset h's price changes on Aggregate macroeconomic shocks as follows:

$$\Delta q_{h,t} = \beta_y \, \Delta y_t + \beta_d \, Asset \, Durability \, Score_{h,t} \times \Delta y_t + \varepsilon_{h,t}. \tag{41}$$

We report our main findings in Table 7. In Specification 1, we observe a positively significant coefficient on aggregate macroeconomic shocks and confirm the procyclical exposure to aggregate fluctuations across assets. Specification 2 shows a positively significant coefficient on the interaction term between asset durability and aggregate shocks. Such a result suggests that assets with higher durability bear higher price fluctuations and thus face significantly higher exposures than those with lower durability to aggregate shocks. As a result, firms hold a basket of assets with higher durability are riskier and earn higher expected returns.

In summary, asset exposures present a positive relation with asset durability to aggregate shocks, which is perfectly consistent with our model implication.

<sup>&</sup>lt;sup>18</sup>Details in price indexes with respect to structures, equipment, and intellectual property product refer to NIPA Table 5.4.4, 5.5.4, and 5.6.4 (https://apps.bea.gov/iTable/iTable.cfm?reqid=19&step=2).

<sup>&</sup>lt;sup>19</sup>Price and GDP changes are deflated by CPI index in real terms.

# 6.2 Asset Pricing Factor Test

In this subsection, we investigate the extent to which the variation in the average returns of the durability-sorted portfolios can be explained by exposure to standard risk factors proposed by the Fama and French (2015) five-factor model, the Hou, Xue, and Zhang (2015) q-factor model, or, more importantly, the collateralizability premium documented in Ai, Li, Li, and Schlag (2019).<sup>20</sup>

To test the standard risk factor models, we preform time-series regressions of asset durability-sorted portfolios' excess returns on the Fama and French (2015) five-factor model (the market factor-MKT, the size factor-SMB, the value factor-HML, the profitability factor-RMW, the investment factor-CMA), and the long-short portfolio sorted on collateralizability (COL) in Panel A and on the Hou, Xue, and Zhang (2015) q-factor model (the market factor-MKT, the size factor-SMB, the investment factor-I/A, the profitability factor-ROE), and the long-short portfolio sorted on collateralizability (COL) in Panel B, respectively. Such time-series regressions enable us to estimate the betas (i.e., risk exposures) of each portfolio's excess return on various risk factors and to estimate each portfolio's risk-adjusted return (i.e., alphas in %). We annualize the excess returns and alphas in Table 8.

# [Place Table 8 about here]

As we show in Table 8, the risk-adjusted returns (intercepts) of the asset durability sorted high-minus-low portfolio remain large and significant, ranging from 8.14% for the Fama and French (2015) five-factor model in Panel A to 8.54% for the Hou, Xue, and Zhang (2015) q-factor model in Panel B, and these intercepts are at least 3.38 standard errors above zero, which the t-statistics is far above 1% statistical significance level. Second, the alpha implied by the Fama-French five-factor model or by the HXZ q-factor model remain comparable to the durability spread (i.e., the return on the high-minus-low portfolio) in the univariate sorting (Table 3). Third, the return on the high-minus-low portfolio has significantly negative market betas with respect to both the Fama and French (2015) five-factor model and to the Hou, Xue, and Zhang (2015) q-factor model; however, the return on the low-minus-high portfolio has insignificantly negative betas with respect to both the Fama and French (2015) five-factor model and to the Hou, Xue, and Zhang (2015) q-factor model. Finally, the asset durability spread cannot be explained by collateralizability (COL), given that asset durability is higher associated with asset collateralizability.

<sup>&</sup>lt;sup>20</sup>The Fama and French factors are downloaded from Kenneth French's data library (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\_library.html). We thank Kewei Hou, Chen Xue, and Lu Zhang for kindly sharing the Hou, Xue, and Zhang factors.

In summary, results from asset pricing tests in Table 8 suggest that the cross-sectional return spread across portfolios sorted on durability cannot be explained by either the Fama and French (2015) five-factor model, the HXZ q-factor model (Hou, Xue, and Zhang (2015)), or the collaterizability prmium. Hence, common risk factors cannot explain the higher returns associated with asset durability. In the following subsection, we reassure the asset durability-return relation by running Fama-Macbeth regressions to control a bundle of firm characteristics.

# 6.3 Fama-Macbeth Regressions

In Section 6.3, we investigate the joint link between the firm-level asset durability and future stock returns using Fama and MacBeth (1973) regressions at firm-level as a valid cross-check the results and establish the robustness of the findings. For robustness, we also investigate the predictive ability of durability for the cross-sectional stock returns using Fama-MacBeth cross-sectional regressions (Fama and MacBeth (1973)). This analysis allows us to control for an extensive list of firm characteristics that predict stock returns and to verify whether the positive durability-return relation is driven by other known predictors at the firm level. This approach is preferable to the portfolio tests, as the latter requires the specific breaking points to sort firms into portfolios and also requires us to select the number of portfolios. Also, it is difficult to include multiple sorting variables with unique information about future stock returns by using a portfolio approach. Thus, Fama-MacBeth cross-sectional regressions provide a reaonable cross-check.

Specifically, we run a Fama-MacBeth firm-level stock return predictability regressions on lagged firm-level asset durability and a list of control variables for other characteristics. The specification of regression is as follows:

$$R_{i,t+1} - R_{f,t+1} = a_j + b \times Asset \ Durability_{i,t} + c \times Leverage_{i,t}d \times Controls_{i,t} + \varepsilon_{i,t}(42)$$

Following Fama and French (1992), we take each month from July of year t to June of year t+1, and we regress monthly returns of individual stock returns (annualized by multiplying 12) on asset durability of year t-1, different sets of control variables that are known by the end of June of year t, and industry fixed effects. Control variables include the natural logarithm of market capitalization at the end of each June (Size) deflated by the CPI index, the natural logarithm of book-to-market ratio (B/M), investment rate (I/A), profitability (ROA), organization capital ratio (OC/AT), R&D intensity (R&D/AT), and industry dummies based on NAICS 3-digit industry classifications. All independent variables are normalized to a zero

mean and a one standard deviation after winsorization at the 1th and 99th percentile to reduce the impact of outliers; we also adjust all independent variables for standard errors by Newey-West adjustment.

### [Place Table 9 about here]

In Table 9, we report the results from cross-sectional regressions performed at a monthly frequency. The reported coefficient is the average slope from monthly regressions, and the corresponding t-statistic is the average slope divided by its time-series standard error. We annualize the slopes and standard errors in Table 9.

The results of Fama-Macbeth regression are consistent with the results of portfolio sorted on durability. To alleviate the confounding effect of levered position, we control for the firm-level leverage ratio in each specification. In Specification 1, asset durability significantly and positively predicts future stock returns with a slope coefficient of 1.46, which is 3.62 standard errors from zero. This finding assures that the asset durability-return relation is mainly driven the leverage channel. In Specification 2, we introduce firm-level collateralizability, according to Ai, Li, Li, and Schlag (2019). In Specification 2, we show that the slope of coefficient on durability remains significant and even larger in magnitude, after explicitly controlling for firm-level collateralizability. In contrast, the coefficient on collateralizability is comparable with the that on durability but with a negative sign. On top of that, Specification 3 highlights that the predictability of asset durability is not subsumed by known predictors for stock returns in the literature, when we put all control variables together to run a horse racing test.

As a whole, Table 9 suggests that the positive asset durability-return relation cannot be attributed to other known predictors and have an unique return predictive power.

# 6.4 Cash Flow Sensitivities of Asset Durability-Sorted Portfolios

Our theory suggests that the asset durability premium comes from different cyclicality of the prices of durable versus less durable capital. In our model, household does not directly trade stocks, therefore, differences in expected returns on the firm's equity must attribute to the differences in the cash flow accruing to entrepreneurs. In this subsection, we measure the cash flow to equity holders and show empirically at the portfolio level that the equity cash flows of firms with high asset durability exhibit a higher, i.e. more positive, sensitivity with respect to two alternative proxies for aggregate macroeconomic shocks: the log difference

(i.e., the growth rate) in TFP and GDP.<sup>21</sup>

According to Belo, Li, Lin, and Zhao (2017), we first aggregate cash flow (represented by EBIT) across the firms in a given portfolio and then normalize this sum by the total lagged sales of that portfolio, and then compute the sensitivity (i.e., loading) of the cash flow with respect to the two aggregate macroeconomic shocks.<sup>22</sup> The results are reported in Table 10.

### [Place Table 10 about here]

Table 10 shows the cash flow sensitivity with respect to TFP or GDP shocks. First, the cash flow sensitivities of asset durability-sorted portfolios display a increasing patter pattern from the lowest to the highest portfolios, ranging from 1.16 (1.33) to 1.78 (1.21) with respect to TFP (GDP) shocks. The loading on the highest quintile portfolio is statistically significant and larger than that of the lowest quintile portfolio. In particular, the difference in TFP (GDP) shock sensitivities between the two extreme portfolios has a t-statistic of 4.25 (2.59). Such a finding again highlights the main economic mechanism in our paper that low durability provides an insurance against aggregate shocks.

#### 6.5 Market Price of Macroeconomic Shocks

Firms with different asset durability differ in their exposures to aggregate macroeconomic shocks and their risk premia. In this subsection, we show that aggregate macroeconomic shock is a source of systematic risk and that exposures to this shock drives the cross-sectional variation of the asset durability sorted portfolios. Consistent with our model, we do so by investigating a two-factor model where the market excess return is the first factor and the macroeconomic shock is the second by estimating the market price of these two factors.

We estimate the parameters of the stochastic discount factor using the generalized method of moments (GMM). The moment restrictions on the excess rate of return of any asset is priced according to the Euler equation. Specifically, the resulting moment restrictions are

$$E[MR_i^e] = 0. (43)$$

In our estimation, we use portfolio returns in excess of risk free rate  $R_i^e$ , so the mean of

<sup>&</sup>lt;sup>21</sup>The data on utilization adjusted total factor productivity (TFP) and GDP are from the Federal Reserve Bank of San Francisco (https://www.frbsf.org/economic-research/indicators-data/total-factor-productivity-tfp/).

<sup>&</sup>lt;sup>22</sup>For robustness, we replace the normalization to total sales and report the sensitivity with respect to two aggregate macroeconomic shocks. The result is indifferent to the normalization and remains consistent with the finding in Panel A of Table 10.

the stochastic discount factor M is not identified from the moment restrictions in equation (43).<sup>23</sup> As the result, we normalize E[M] = 1. Given this normalization, we can rearrange the moment condition in the above equation as

$$E[R_i^e] = -Cov(M, R_i^e), \tag{44}$$

which is the empirical equivalent to our model, but with the conditional moments replaced by their unconditional counterparts. We assess the model's ability to price test assets correctly on the basis of residuals of the Euler equation (44).

The empirical equivalent of the stochastic discount factor (SDF) in our model denotes as

$$M_t = 1 - b_M \times \text{MKT}_t - b_A \times \text{Macro}_t, \tag{45}$$

which specifies that investors' marginal utility is driven by two aggregate shocks, MKT<sub>t</sub>, which is spanned by the market factor in the standard capital asset pricing model (CAPM), and Macro<sub>t</sub>, which is the aggregate macroeconomic shock. We take the log difference in wealth share and TFP to proxy for the aggregate macroeconomic shock. We compute the sum of squared errors (SSQE) and the *J*-statistic of the overidentifying restrictions of the model. That is, all the pricing errors are zero if our model specification is correct. Finally, we report two-setp GMM estimates of  $b_M$  and  $b_A$  using the identity matrix to weigh moment restrictions, and adjust the standard errors using the Newey-West procedure with a maximum of three lags.

#### [Place Table 11 about here]

Panel A of Table 11 presents the average excess returns and risk characteristics for the five portfolios of firms sorted on their asset durability portfolios. First, the sensitivity with respect to the TFP (GDP) shock display a largely upward-sloping pattern from the lowest to the highest quintile portfolio and the long-short portfolio. These portfolios present a upward-sloping pattern of covariances with the empirical measures of the aggregate macroeconomic shock. Namely, the highest asset durability quintile faces the highest risk exposure and thus exhibits higher sensitivity than the lowest asset durability quintile with respect to aggregate macroeconomic shocks. Second, the difference in sensitivities between two extreme portfolios (i.e., the lowest and the highest portfolio) is positively significant with a t-statistic of 2.15 and 1.85, depending on whether the aggregate macroeconomic shock is measured as the TFP

<sup>&</sup>lt;sup>23</sup>Given that our testing assets are portfolio returns in excess of the risk-free rate, the mean of the SDF is not identified. Without loss of generality, we take a normalization E[M] = 1, which leads the moment condition in equation (44). Details refer to Cochrane (2005), page 256-257.

or GDP shock.

Panel B of Table 11 presents results using the five asset durability-sorted portfolios. The estimates of the price of risk of the aggregate macroeconomic shock are statistically significant across specifications, ranging from 0.24 to 0.70 when using the the log difference TFP or GDP. In terms of asset pricing errors, including measures of the aggregate macroeconomic shock improves upon the ability of CAPM to price the cross-section of asset durability-sorted portfolios, reducing the sum of squares to 0.1-0.39 relative to 0.78 and the mean absolute pricing errors to 1.17-2.30 relative to 2.72 when using difference measures of the aggregate macroeconomic shock. Last, the *J*-test is statistically insignificant and does not reject the model when we introduce the two-factor model, which implies that the average pricing error becomes smaller and even statistically insignificant. Therefore, the two-factor model is sufficient to capture the cross-sectional variations in the asset durability-sorted portfolios.

# 7 Conclusion

In this paper, we present a general equilibrium asset pricing model with heterogeneous firms and collateral constraints. Our model predicts that the price of durable asset features higher cyclicality, faces more exposures to aggregate shocks, and, therefore, earns a higher expected return, since firms choose to hold a lower fraction of durable assets to relax the collateral constraint, when their constraint is more binding in recessions than in booms.

We develop a novel measure of the asset durability from firms' assets and document empirical findings consistent with our model predictions. In particular, we find that a significant return spread between firms with a high asset durability versus a low asset durability amounts to 5% per year. When we calibrate our model to the dynamics of macroeconomic quantities, we show that the credit market friction channel is a quantitatively important determinant for the cross-sectional stock returns.

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Table 1: Durability and Financial Constraints

This table shows the coefficients of regressions of asset durability on various financial constraints (controlling for industry dummies at NAICS 3-digit Code level). A detailed definition of the variables refers to Table C.3. All independent variables are normalized to zero mean and one standard deviation after winsorization at the 1th and 99th percentile of their empirical distribution. We include t-statistics in parentheses. The sample excludes utility, financial, public administrative, and public administrative industries, and starts from 1977 to 2016.

| Variables    | (1)     | (2)     | (3)         | (4)     | (5)    | (6)    | (7)    |
|--------------|---------|---------|-------------|---------|--------|--------|--------|
| Non-DIV      | -1.75   |         |             |         | -0.80  |        |        |
| [t]          | 14.64   |         |             |         | 10.55  |        |        |
| SA           |         | -1.47   |             |         |        | -1.42  |        |
| [t]          |         | -20.10  |             |         |        | -13.66 |        |
| WW           |         |         | -1.08       |         |        |        | -1.10  |
| [t]          |         |         | -13.72      |         |        |        | -11.95 |
| ROA          |         |         |             | 1.07    | 0.68   | 0.61   | 0.69   |
| [t]          |         |         |             | 15.00   | 9.70   | 8.93   | 9.38   |
| Log ME       |         |         |             |         | 0.11   | -0.84  | -0.80  |
| [t]          |         |         |             |         | 1.73   | -8.43  | -10.23 |
| Log B/M      |         |         |             |         | 0.38   | -0.04  | 0.03   |
| [t]          |         |         |             |         | 8.25   | -0.64  | 0.58   |
| I/K          |         |         |             |         | -0.58  | -0.51  | -0.53  |
| [t]          |         |         |             |         | -9.03  | -8.56  | -8.46  |
| Lev.         |         |         |             |         | 0.73   | -0.41  | -0.27  |
| [t]          |         |         |             |         | 3.33   | -1.64  | -1.04  |
| Cash/AT      |         |         |             |         | 0.45   | 0.48   | 0.48   |
| [t]          |         |         |             |         | 4.30   | 4.68   | 4.50   |
| Redp         |         |         |             |         | -0.10  | -0.08  | -0.11  |
| [t]          |         |         |             |         | -0.34  | -0.27  | -0.34  |
| TANT         |         |         |             |         | 3.83   | 3.88   | 3.84   |
| [t]          |         |         |             |         | 17.00  | 17.33  | 17.05  |
| Observations | 130,059 | 130,059 | $120,\!135$ | 129,924 | 99,292 | 99,292 | 94,299 |
| R-squared    | 0.48    | 0.50    | 0.50        | 0.49    | 0.68   | 0.69   | 0.69   |
| Controls     | No      | No      | No          | No      | Yes    | Yes    | Yes    |
| Time FE      | Yes     | Yes     | Yes         | Yes     | Yes    | Yes    | Yes    |
| Industry FE  | Yes     | Yes     | Yes         | Yes     | Yes    | Yes    | Yes    |
| Cluster SE   | Yes     | Yes     | Yes         | Yes     | Yes    | Yes    | Yes    |

#### Table 2: Summary Statistics

This table presents summary statistics for the main outcome variables and control variables of our sample. The detailed definition of asset durability and depreciation measure refers to Section 2.1 Debt leverage is the ratio of long-term debt (DLTT) over the sum of leased capital and total assets (AT), where leased capital is defined as 10 times rental expense (XRENT). Rental leverage is the ratio of leased capital over the sum of leased capital and total assets (AT). Leased capital leverage is the sum of debt leverage and rental leverage. In Panel A, we split the whole sample into constrained and unconstrained firms at the end of every June, as classified by dividend payment dummy (DIV), according to Farre-Mensa and Ljungqvist (2016). We report pooled means of these variables value-weighted by firm market capitalization at fiscal year end. In Panel B, we report the time-series averages of the cross-sectional median of firm characteristics across five portfolios sorted on asset durability relative to their industry peers according to the NAICS 3-digit industry classifications. The detailed definition of the variables is listed in Appendix C. The sample is 1977 to 2016 and excludes financial, utility, and public administrative from the analysis.

|                                   | Panel A: Pooled Statistics |                       | Panel B: Firm Characteristics |                      |                       |                       |                       |
|-----------------------------------|----------------------------|-----------------------|-------------------------------|----------------------|-----------------------|-----------------------|-----------------------|
|                                   | Const. Unconst.            |                       | Portfolios                    |                      |                       |                       |                       |
| Variables                         |                            | Mean                  | L                             | 2                    | 3                     | 4                     | Н                     |
| Durability Depreciation Book Lev. | 12.66<br>0.17<br>0.24      | 16.54<br>0.13<br>0.33 | 7.69<br>0.19<br>0.13          | 9.99<br>0.16<br>0.19 | 11.45<br>0.15<br>0.21 | 14.24<br>0.13<br>0.28 | 18.00<br>0.11<br>0.32 |

Table 3: Portfolios Sorted on Asset Durability

This table shows average excess returns for five portfolios sorted on asset durability across firms relative to their industry peers, for which we use the NAICS 3-digit industry classifications and rebalance portfolios at the end of every June. The results reflect monthly data, for which the sample is from July 1978 to December 2017 and excludes utility, financial, public administrative, and public administrative industries. We split the whole sample into financially constrained and unconstrained subsample at the end of every June, as classified by dividend payment dummy, SA index, rating dummy, and WW index. We report average excess returns over the risk-free rate E[R]- $R_f$ , standard deviations Std, and Sharpe ratios SR across five portfolios in constrained subsamples (Panel A) and in whole sample (Panel B). Standard errors are estimated by using the Newey-West correction. We include t-statistics in parentheses and annualize portfolio returns multiplying by 12. All returns, standard deviations, and Sharpe ratios have been annualized.

|                          | Panel A: Constrained Subsample |       |        |         |       |       |
|--------------------------|--------------------------------|-------|--------|---------|-------|-------|
|                          | L                              | 2     | 3      | 4       | Н     | H-L   |
|                          |                                |       | D      | IV      |       |       |
| E[R]-R <sub>f</sub> (%)  | 5.39                           | 9.57  | 9.34   | 9.03    | 12.32 | 6.93  |
| [t]                      | 1.48                           | 2.81  | 2.81   | 2.92    | 3.62  | 2.86  |
| Std (%)                  | 26.79                          | 25.32 | 24.81  | 24.05   | 24.09 | 11.80 |
| SR                       | 0.20                           | 0.38  | 0.38   | 0.38    | 0.51  | 0.59  |
|                          |                                |       | SA I   | ndex    |       |       |
| E[R]-R <sub>f</sub> (%)  | 4.53                           | 7.59  | 7.97   | 8.39    | 9.63  | 5.10  |
| [t]                      | 1.12                           | 1.89  | 1.98   | 2.35    | 2.77  | 2.54  |
| Std (%)                  | 24.45                          | 23.55 | 24.34  | 21.09   | 20.7  | 11.58 |
| SR                       | 0.19                           | 0.32  | 0.33   | 0.40    | 0.47  | 0.44  |
|                          |                                |       | Rat    | ting    |       |       |
| $E[R]-R_f$ (%)           | 5.65                           | 8.76  | 9.40   | 9.35    | 10.10 | 4.45  |
| [t]                      | 1.42                           | 2.18  | 3.06   | 2.84    | 3.52  | 2.12  |
| $\operatorname{Std}(\%)$ | 24.32                          | 23.4  | 19.61  | 19.89   | 18.81 | 11.8  |
| SR                       | 0.23                           | 0.37  | 0.48   | 0.47    | 0.54  | 0.38  |
|                          |                                |       | ww     | Index   |       |       |
| E[R]-R <sub>f</sub> (%)  | 6.09                           | 8.24  | 9.13   | 9.59    | 9.65  | 3.56  |
| [t]                      | 2.13                           | 2.78  | 3.68   | 3.78    | 3.85  | 2.23  |
| Std (%)                  | 25.7                           | 24.18 | 23.67  | 21.1    | 20.85 | 11.04 |
| SR                       | 0.24                           | 0.34  | 0.39   | 0.45    | 0.46  | 0.32  |
|                          |                                | Pane  | 1 B: W | hole Sa | mple  |       |
| E[R]-R <sub>f</sub> (%)  | 7.36                           | 8.10  | 8.12   | 8.65    | 8.79  | 1.44  |
| [t]                      | 2.70                           | 3.49  | 3.26   | 4.17    | 3.55  | 1.03  |
| Std (%)                  | 19.25                          | 16.75 | 15.14  | 15.15   | 17.37 | 8.72  |
| SR                       | 0.38                           | 0.48  | 0.54   | 0.57    | 0.51  | 0.17  |

Table 4: Calibration

We calibrate the model at the quarterly frequency. This table reports the parameter values and the corresponding moments (annalized) we used in the calibration procedure.

| Parameter                          | Symbol         | Value   |
|------------------------------------|----------------|---------|
| Relative risk aversion             | $\gamma$       | 10      |
| IES                                | $\psi$         | 2       |
| Capital share                      | $\alpha$       | 0.30    |
| Span of control parameter          | ν              | 0.90    |
| Mean productivity growth rate      | $E(\tilde{A})$ | 0.1248  |
| Time discount factor               | $\beta$        | 0.99    |
| Durable capital dep. rate          | $\delta_d$     | 0.01    |
| Non-durable capital dep. rate      | $\delta_{nd}$  | 0.03    |
| Death rate of entrepreneurs        | $E(\lambda)$   | 0.025   |
| Collateralizability parameter      | $\theta$       | 0.33    |
| Transfer to entering entrepreneurs | χ              | 0.89    |
| Persistence of TFP shock           | $ ho_A$        | 0.994   |
| Persistence of $\lambda$ shock     | $ ho_x$        | 0.98    |
| Vol. of $\lambda$ shock            | $\sigma_x$     | 0.05    |
| Vol. of productivity shock         | $\sigma_A$     | 0.00695 |
| Inv. adj. cost parameter           | ζ              | 25      |
| Mean idio. productivity growth     | $\mu_Z$        | 0.005   |
| Vol. of idio. productivity growth  | $\sigma_Z$     | 0.025   |

### Table 5: Model Simulations and Aggregate Moments

This table presents the moments from the model simulation. The market return  $R_M$  corresponds to the return on entrepreneurs' net worth and embodies an endogenous financial leverage.  $R_d^{Lev}$ ,  $R_{nd}^{Lev}$  denotes the levered capital returns, by the average financial leverage in the economy. We simulate the economy at monthly frequency, then aggregate the monthly observations to annual frequency. The moments reported are based on the annual observations. Number in parenthesis are standard errors of the calculated moments.

| Moments                    | Data        | Model |
|----------------------------|-------------|-------|
| $\sigma(\Delta y)$         | 3.05 (0.60) | 3.32  |
| $\sigma(\Delta c)$         | 2.53 (0.56) | 2.88  |
| $\sigma(\Delta i)$         | 10.30(2.36) | 6.15  |
| $corr(\Delta c, \Delta i)$ | 0.39(0.29)  | 0.77  |
| $AC1(\Delta c)$            | 0.49(0.15)  | 0.45  |
| $E[R_M - R_f]$             | 5.71 (2.25) | 6.82  |
| $\sigma(R_M - R_f)$        | 20.89(2.21) | 16.04 |
| $E[R_f]$                   | 1.10(0.16)  | 1.15  |
| $\sigma(R_f)$              | 0.97(0.31)  | 0.80  |
| $E[R_d^{Lev} - R_f]$       |             | 5.50  |
| $E[R_{nd}^{Lev} - R_f]$    |             | 1.50  |

#### Table 6: Asset Durability Spread, Data, and Model Comparison

This table compares the moments in the empirical data (Panel A) and the model simulated data (Panel B) at the portfolio level. Panel A reports the statistics computed from the sample of financially constrained firms in the data, as classified by dividend payment dummy (DIV). In Panel B, we implement model simulation and then perform the same portfolio sorts as in the data. Panel A and B show the time series average of the cross-sectional median of firm characteristics using the value from the year end, including asset durability, depreciation rate, book leverage, return on equity. We also report the value-weighted excess returns E[R]- $R_f(\%)$  (annualized by multiplying by 12, in percentage terms), for quintile portfolios sorted on asset durability. The detailed definition of the variables is listed in Appendix C. The sample is from July 1978 to December 2017 and excludes financial, utility, and public administrative industries from the analysis.

| Variables          | L             | 2     | 3       | 4       | Н     | H-L  |  |
|--------------------|---------------|-------|---------|---------|-------|------|--|
|                    | Panel A: Data |       |         |         |       |      |  |
| Asset Durability   | 7.69          | 9.99  | 11.45   | 14.24   | 18.00 |      |  |
| Depreciation       | 0.19          | 0.16  | 0.15    | 0.13    | 0.11  |      |  |
| Book Lev.          | 0.13          | 0.19  | 0.21    | 0.28    | 0.32  |      |  |
| ROE                | 0.12          | 0.17  | 0.18    | 0.22    | 0.23  |      |  |
| $E[R]$ - $R_f$ (%) | 5.39          | 9.57  | 9.34    | 9.03    | 12.32 | 6.93 |  |
|                    |               | ]     | Panel E | 3: Mode | el    |      |  |
| Asset Durability   | 8.33          | 10.05 | 11.12   | 14.28   | 20.08 |      |  |
| Depreciation       | 0.12          | 0.10  | 0.09    | 0.07    | 0.05  |      |  |
| Book Lev.          | 0.19          | 0.27  | 0.33    | 0.39    | 0.45  |      |  |
| ROE                | 0.06          | 0.08  | 0.09    | 0.11    | 0.13  |      |  |
| $E[R]$ - $R_f$ (%) | 3.39          | 5.27  | 5.96    | 6.60    | 7.02  | 3.63 |  |

#### Table 7: Aggregate Shocks and Price Dynamics

This table shows the exposure of price dynamics to aggregate macroeconomic shocks. All estimates are based on the following panel regressions:

$$\Delta q_{h,t} = \beta_y \ \Delta y_t + \beta_d \ Asset \ Durability_{h,t} \times \Delta y_t + \varepsilon_{h,t},$$

in which  $\Delta q_{h,t}$  denotes price dynamics of asset h,  $\Delta y_t$  denotes aggregate macroeconomic shocks, and Asset Durability h denotes the asset durability of asset h at year t. We control for asset fixed effects, and standard errors are clustered at the asset level. We report t-statistics in parenthesis. The sample period is from 1977 to 2017.

|  | (1)                 | (2)                          |
|--|---------------------|------------------------------|
| $\begin{array}{c} \mathrm{dy} \\ [t] \\ \mathrm{Interaction} \\ [t] \end{array}$ | 1.51<br>11.71       | 1.02<br>3.89<br>1.06<br>3.28 |
| Observations<br>Asset FE<br>Cluster SE   | 4,830<br>Yes<br>Yes | 4,760<br>Yes<br>Yes          |

Table 8: Asset Pricing Factor Tests

This table shows asset pricing test for five portfolios sorted on asset durability across firms relative to their industry peers, where we use the NAICS 3-digit industry classifications and rebalance portfolios at the end of every June. The results reflect monthly data, for which the sample is from July 1978 to December 2017 and excludes utility, financial, and public administrative industries. We split the whole sample into financially constrained and unconstrained firms, as classified by the dividend payment dummy (DIV), and report five portfolios across the financially constrained subsample. In Panel A, we report the portfolio alphas and betas by the Fama-French five-factor model plus the long-short portfolio sorted on collateralizability (COL), including MKT, SMB, HML, RMW, CMA, and LMH. In panel B, we report portfolio alphas and betas by the HXZ q-factor model plus the long-short portfolio sorted on collateralizability, including MKT, SMB, I/A, ROE, and COL. Data on the Fama-French five-factor model are from Kenneth French's website. Data on the I/A and ROE factor are provided by Kewei Hou, Chen Xue, and Lu Zhang. Data on the long-short portfolio sorted on collateralizability refers to Ai, Li, Li, and Schlag (2019). Standard errors are estimated using Newey-West correction. We include t-statistics in parentheses and annualize the portfolio alphas by multiplying 12.

|                          | ${f L}$              | 2     | 3       | 4      | Н     | H-L   |  |  |
|--------------------------|----------------------|-------|---------|--------|-------|-------|--|--|
|                          | Panel A: $FF5 + LMH$ |       |         |        |       |       |  |  |
| $\alpha_{	ext{FF5+COL}}$ | -4.13                | 2.51  | 1.55    | 0.43   | 4.02  | 8.14  |  |  |
| [t]                      | -2.06                | 1.44  | 0.94    | 0.29   | 2.52  | 3.38  |  |  |
| MKT                      | 1.28                 | 1.14  | 1.15    | 1.13   | 1.17  | -0.11 |  |  |
| [t]                      | 24.57                | 32.69 | 29.01   | 36.65  | 33.10 | -2.22 |  |  |
| SMB                      | 0.51                 | 0.46  | 0.36    | 0.46   | 0.43  | -0.08 |  |  |
| [t]                      | 5.97                 | 6.35  | 6.22    | 8.25   | 7.54  | -0.91 |  |  |
| HML                      | -0.24                | -0.35 | -0.33   | -0.46  | -0.38 | -0.15 |  |  |
| [t]                      | -2.45                | -4.77 | -4.35   | -6.83  | -4.92 | -1.69 |  |  |
| RMW                      | -0.10                | -0.24 | -0.11   | 0.02   | -0.06 | 0.04  |  |  |
| [t]                      | -0.78                | -2.19 | -1.53   | 0.34   | -0.78 | 0.25  |  |  |
| CMA                      | -0.44                | -0.42 | -0.51   | -0.31  | -0.25 | 0.19  |  |  |
| [t]                      | -3.21                | -4.18 | -4.58   | -3.27  | -2.88 | 1.47  |  |  |
| COL                      | 0.10                 | 0.13  | 0.13    | 0.09   | 0.03  | -0.07 |  |  |
| [t]                      | 2.67                 | 3.50  | 3.69    | 2.88   | 0.83  | -1.67 |  |  |
|                          |                      | Pane  | el B: H | XZ + I | LMH   |       |  |  |
| $\alpha_{\rm HXZ+COL}$   | -4.71                | 1.65  | 1.60    | -0.30  | 3.82  | 8.54  |  |  |
| [t]                      | -2.36                | 0.86  | 0.79    | -0.17  | 2.26  | 3.48  |  |  |
| MKT                      | 1.31                 | 1.18  | 1.17    | 1.15   | 1.18  | -0.13 |  |  |
| [t]                      | 19.40                | 28.08 | 26.40   | 28.47  | 30.62 | -2.20 |  |  |
| SMB                      | 0.42                 | 0.37  | 0.26    | 0.37   | 0.37  | -0.06 |  |  |
| [t]                      | 3.30                 | 3.96  | 4.37    | 5.74   | 7.01  | -0.42 |  |  |
| I/A                      | -0.62                | -0.77 | -0.88   | -0.80  | -0.69 | -0.08 |  |  |
| [t]                      | -5.18                | -8.05 | -9.03   | -9.30  | -8.59 | -0.64 |  |  |
| ROE                      | -0.03                | -0.08 | -0.04   | 0.12   | 0.01  | 0.04  |  |  |
| [t]                      | -0.34                | -0.98 | -0.55   | 1.92   | 0.17  | 0.62  |  |  |
| COL                      | 0.17                 | 0.24  | 0.21    | 0.18   | 0.11  | -0.06 |  |  |
| [t]                      | 3.36                 | 6.21  | 6.36    | 6.13   | 3.83  | -1.15 |  |  |

#### Table 9: Fama-Macbeth Regressions

This table reports the of Fama-Macbeth regressions of individual stock excess returns on their asset durability and other firm characteristics. The sample is from July 1978 to December 2017 and excludes financial, utility, and public administrative industries from the analysis. We split the whole sample into financially constrained and unconstrained firms, as classified by the dividend payment dummy, and then report the result of regression in the financially constrained subsample. For each month from July of year t to June of year t+1, we regress monthly excess returns of individual stock on durability with different sets of variables that are known by the end of June of year t, and control for industry fixed effects based on NAIC 3-digit industry classifications. We present the time-series average and heteroscedasticity-robust t-statistics of the slopes (i.e., coefficients) estimated from the monthly cross-sectional regressions for different model specifications. All independent variables are normalized to zero mean and one standard deviation after winsorization at the 1th and 99th percentile of their empirical distribution. We include t-statistics in parentheses and annualize individual stock excess returns by multiplying 12. Standard errors are estimated using Newey-West correction.

| Variables           | (1)     | (2)     | (3)     |
|---------------------|---------|---------|---------|
| Asset Durability    | 2.13    | 3.62    | 1.46    |
| [t]                 | 3.44    | 5.24    | 2.86    |
| Book Lev.           | -1.89   | -0.57   | -0.99   |
| [t]                 | -4.17   | -1.09   | -2.28   |
| Collateralizability |         | -3.07   |         |
| [t]                 |         | -3.87   |         |
| Log ME              |         |         | -0.75   |
| [t]                 |         |         | -0.67   |
| Log B/M             |         |         | 4.82    |
| [t]                 |         |         | 8.73    |
| ROA                 |         |         | 6.36    |
| [t]                 |         |         | 8.98    |
| I/K                 |         |         | -1.13   |
| [t]                 |         |         | -2.78   |
| OC/AT               |         |         | 1.03    |
| [t]                 |         |         | 2.29    |
| R&D/AT              |         |         | 5.71    |
| [t]                 |         |         | 7.05    |
| Observations        | 846,277 | 632,464 | 806,449 |
| Controls            | No      | No      | Yes     |
| Industry FE         | Yes     | Yes     | Yes     |

#### Table 10: Cash Flow Sensitivity

This table shows the cash flow sensitivity of the asset durability-sorted portfolios to the TFP and GDP shock. Panel A and B report sensitivities from empirical data and model simulated data, respectively. The portfolio-level normalized cash flow is constructed by aggregating cash flow (EBIT) within each quintile portfolio, and then normalized by the lagged aggregate sales (SALE) of the given portfolio. We regress portfolio-level normalized cash flow on TFP and wealth share shock, respectively, and then report estimated coefficients on normalized cash flow. Standard errors are estimated by Newey-West correction, and t-statistics are included in parentheses. All regressions are conducted at the annual frequency. The sample includes annual data from 1979 to 2017.

|     | L     | 2    | 3     | 4     | Н    | H-L  |
|-----|-------|------|-------|-------|------|------|
| TFP | 1.16  | 1.29 | 1.63  | 1.58  | 1.78 | 0.62 |
| [t] | 14.95 | 8.88 | 17.82 | 10.30 | 9.06 | 4.25 |
| GDP | 1.33  | 2.01 | 2.10  | 2.08  | 2.54 | 1.21 |
| [t] | 3.76  | 5.79 | 4.49  | 4.72  | 4.60 | 5.59 |

#### Table 11: Estimating the Market Price of Risk

This table shows results the GMM estimates of the stochastic discount factor's parameters. In Panel A, we use the asset durability-sorted portfolios as test portfolios and report risk exposures with respect to the measures of aggregate macroeconomic shock. We use two sets of proxies for the aggregate macroeconomic shock (Macro): the the log difference in TFP and GDP. In Panel B, we present GMM estimates of the parameters of the stochastic discount factor  $M = 1 - b_M \text{MKT} - b_A \times \text{Macro}$ , using the leased capital ratio sorted portfolios. We do the normalization such that E[M] = 1 (See, e.g., Cochrane (2005)). We report HAC t-statistics computed errors using the Newey-West procedure adjusted for three lags. As a measure of fit, we report the sum of squared errors (SSQE), mean absolute pricing errors (MAPE), and the J-statistic of the overidentifying restrictions of the model. The sample includes annual data from 1979 to 2017.

| Panel A: Portfolio Risk Exposures |         |      |      |      |      |      |  |
|-----------------------------------|---------|------|------|------|------|------|--|
|                                   | ${f L}$ | 2    | 3    | 4    | Н    | H-L  |  |
| TFP                               | 0.36    | 1.92 | 1.37 | 1.48 | 2.33 | 1.89 |  |
| [t]                               | 0.75    | 1.93 | 1.34 | 1.73 | 2.16 | 2.15 |  |
| GDP                               | -0.09   | 2.97 | 1.63 | 1.48 | 3.32 | 3.37 |  |
| [t]                               | -0.03   | 0.83 | 0.51 | 0.37 | 0.75 | 1.85 |  |

| Panel B: Price of Risks |      |      |      |  |  |  |  |
|-------------------------|------|------|------|--|--|--|--|
| Parameters              | CAPM | TFP  | GDP  |  |  |  |  |
| $\overline{b_M}$        | 0.02 | 0.01 | 0.01 |  |  |  |  |
| [t]                     | 3.66 | 1.84 | 1.17 |  |  |  |  |
| $b_A$                   |      | 0.24 | 0.70 |  |  |  |  |
| [t]                     |      | 7.78 | 4.95 |  |  |  |  |
| SSEQ (%)                | 0.78 | 0.10 | 0.39 |  |  |  |  |
| MAPE $(\%)$             | 2.72 | 1.17 | 2.30 |  |  |  |  |
| J-test                  | 6.69 | 3.13 | 3.40 |  |  |  |  |
| p                       | 0.24 | 0.53 | 0.49 |  |  |  |  |

# Appendix A: Proof of Proposition 1

We prove Proposition 1 in two steps: first, given prices, the quantities satisfy the household's and the entrepreneurs' optimality conditions; second, the quantities satisfy the market clearing conditions.

To verify the optimality conditions, note that the optimization problems of households and firms are all standard convex programming problems; therefore, we only need to verify first order conditions. Equation (28) is the household's first-order condition. Equation (34) is a normalized version of resource constraint (15). Both of them are satisfied as listed in Proposition 1.

To verify that the entrepreneur i's allocations  $\{N_{i,t}, B_{i,t}, K_{i,t}^d, K_{i,t}^{nd}, L_{i,t}\}$  as constructed in Proposition 1 satisfy the first order conditions for the optimization problem in equation (8), note that the first order condition with respect to  $B_{i,t}$  implies

$$\mu_t^i = E_t \left[ \widetilde{M}_{t+1} \right] R_t^f + \eta_t^i. \tag{A1}$$

Similarly, the first order condition for type-d capital  $K_{i,t+1}^d$  is

$$\mu_t^i = E_t \left[ \widetilde{M}_{t+1}^i \frac{\prod_{K^d} \left( \bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}^d, K_{i,t+1}^{nd} \right) + (1 - \delta_d) \, q_{d,t+1}}{q_{d,t}} \right] + \theta \eta_t^i. \tag{A2}$$

Finally, the optimality with respect to the choice of type-nd capital  $K_{i,t+1}^{nd}$  implies

$$\mu_t^i = E_t \left[ \widetilde{M}_{t+1}^i \frac{\prod_{K^{nd}} \left( \bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}^d, K_{i,t+1}^{nd} \right) + (1 - \delta_{nd}) q_{nd,t+1}}{q_{d,t}} \right] + \theta \eta_t^i.$$
 (A3)

Next, the law of motion of the endogenous state variable n can be constructed from equation (9):

$$n' = (1 - \lambda) \begin{bmatrix} \alpha \nu A' + \zeta (1 - \delta_d) q_d (A', n') + (1 - \zeta) (1 - \delta_{nd}) q_{nd} (A', n') \\ -\theta [\zeta q_d (A, n) + (1 - \zeta) q_{nd} (A, n)] R_f (A, n) \end{bmatrix} + \lambda \chi \frac{n}{\Gamma (A, n)}.$$
(A4)

With the law of motion of the state variables, we can construct the normalized utility of the

household as the fixed point of

$$u(A,n) = \left\{ (1-\beta)c(A,n)^{1-\frac{1}{\psi}} + \beta\Gamma(A,n)^{1-\frac{1}{\psi}} \left( E[u(A',n')^{1-\gamma}] \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}}.$$

The stochastic discount factors must be consistent with household utility maximization:

$$M' = \beta \left[ \frac{c(A', n') \Gamma(A, n)}{c(A, n)} \right]^{-\frac{1}{\psi}} \left[ \frac{u(A', n')}{E \left[ u(A', n')^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right]^{\frac{1}{\psi} - \gamma}$$
(A5)

$$\widetilde{M}' = M'[(1-\lambda)\mu(A',n') + \lambda]. \tag{A6}$$

In our setup, thanks to the assumptions that the idiosyncratic shock  $z_{i,t+1}$  is observed before the decisions on  $K^d_{i,t+1}$  and  $K^{nd}_{i,t+1}$  are made, we can construct an equilibrium in which  $\mu^i_t$  and  $\eta^i_t$  are equalized across all the firms because  $\frac{\partial}{\partial K^d_{i,t+1}} \Pi\left(\bar{A}_{t+1}, z_{i,t+1}, K^d_{i,t+1}, K^{nd}_{i,t+1}\right) = \frac{\partial}{\partial K^{nd}_{i,t+1}} \Pi\left(\bar{A}_{t+1}, z_{i,t+1}, K^d_{i,t+1}, K^{nd}_{i,t+1}\right)$  are the same for all i.

Our next step is to verify the market clearing conditions. Given the initial conditions (initial net worth  $N_0$ ,  $\frac{K_1^d}{K_1^{nd}} = \frac{\zeta}{1-\zeta}$ ,  $N_{i,0} = z_{i,1}N_0$ ) and the net worth injection rule for the new entrant firms  $(N_{t+1}^{entrant} = \chi N_t$  for all t), we establish the market clearing conditions through the following lemma. For simplicity, we assume the collateral constraint to be binding. The case in which this constraint is not binding can be dealt with in a similar way.

**Lemma 1.** The optimal allocations  $\{N_{i,t}, B_{i,t}, K_{i,t+1}^d, K_{i,t+1}^{nd}\}$  constructed as in Proposition 1 satisfy the market clearing conditions, i.e.,

$$K_{t+1}^d = \int K_{i,t+1}^d di, \quad K_{t+1}^{nd} = \int K_{i,t+1}^{nd} di, \quad N_t = \int N_{i,t} di$$
 (A7)

for all  $t \geq 0$ .

First, in each period t, given prices and  $N_{i,t}$ , the individual entrepreneur i's capital decisions  $\{K_{i,t+1}^d, K_{i,t+1}^{nd}\}$  must satisfy the condition

$$N_{i,t} = [1 - \theta] q_{d,t} K_{i,t+1}^d + [1 - \theta] q_{nd,t} K_{i,t+1}^{nd}$$
(A8)

and the optimal decision rule (19). Equation (A8) is obtained by combining the entrepreneur's budget constraint (4) with a binding collateral constraint (5).

Next, we show by induction, that, given the initial conditions, market clearing conditions (A7) hold for all  $t \geq 0$ . In period 0, we start from the initial conditions. First,  $N_{i,0} = z_{i,1}N_0$ , where  $z_{i,1}$  is chosen from the stationary distribution of z. Then, given  $z_{i,1}$  for each firm

i, we use equations (A8) and (19) to solve for  $K_{i,1}^d$  and  $K_{i,1}^{nd}$ . Clearly,  $K_{i,1}^d = z_{i,1}K_1^d$  and  $K_{i,1}^{nd} = z_{i,1}K_1^{nd}$ . Therefore, the market clearing conditions (A7) hold for t = 0, i.e.,

$$\int K_{i,1}^d di = K_1^d, \quad \int K_{i,1}^{nd} di = K_1^{nd}, \quad \int N_{i,0} di = N_0.$$
 (A9)

To complete the induction argument, we need to show that if market clearing holds for t+1, it must hold for t+2 for all t, which is the following claim:

Claim 1. Suppose  $\int K_{i,t+1}^d di = K_{t+1}^d$ ,  $\int K_{i,t+1}^{nd} di = K_{t+1}^{nd}$ ,  $\int N_{i,t} di = N_t$ , and  $N_{t+1}^{entrant} = \chi N_t$ , then

$$\int K_{i,t+2}^d di = K_{t+2}^d \quad \int K_{i,t+2}^{nd} di = K_{t+2}^{nd} \quad \int N_{i,t+1} di = N_{t+1}$$
 (A10)

for all  $t \geq 0$ .

1. Using the law of motion for the net worth of existing firms, one can show that the total net worth of all surviving firms can be rewritten as follows:

$$(1 - \lambda) \int N_{i,t+1} di$$

$$= (1 - \lambda) \int \begin{bmatrix} A_{t+1} \left( K_{i,t+1}^d + K_{i,t+1}^{nd} \right) + (1 - \delta_d) q_{d,t+1} K_{i,t+1}^d \\ + (1 - \delta_{nd}) q_{nd,t+1} K_{i,t+1}^{nd} - R_{f,t} B_{i,t} \end{bmatrix} di,$$

$$= (1 - \lambda) \left[ A_{t+1} \left( K_{t+1}^d + K_{t+1}^{nd} \right) + (1 - \delta_d) q_{d,t} K_{t+1}^d + (1 - \delta_{nd}) q_{nd,t} K_{t+1}^{nd} - R_{f,t} B_t \right],$$

since by assumption  $\int K_{i,t+1}^d di = K_{t+1}^d$ ,  $\int K_{i,t+1}^{nd} di = K_{t+1}^{nd}$ , and  $\int B_{i,t} di = B_t = \theta \left[ q_{d,t} K_{t+1}^d + q_{nd,t} K_{t+1}^{nd} \right]$ . Using the assignment rule for the net worth of new entrants,  $N_{t+1}^{entrant} = \chi N_t$ , we can show that the total net worth at the end of period t+1 across survivors and new entrants together satisfies  $\int N_{i,t+1} di = N_{t+1}$ , where aggregate net worth  $N_{t+1}$  is given by equation (9).

2. At the end of period t+1, we have a pool of firms consisting of old ones with net worth given by (7) and new entrants. All of them will observe  $z_{i,t+2}$  (for the new entrants  $z_{i,t+2} = \bar{z}$ ) and produce at the beginning of the period t+1.

We compute the capital holdings for period t+2 for each firm i using (A8) and (19). At this point, the capital holdings and the net worth of all existing firms will not be proportional to  $z_{i,t+2}$  due to heterogeneity in the shocks. However, we know that  $\int N_{i,t+1} di = N_{t+1}$ , and  $\int z_{i,t+2} di = 1$ . Integrating (A8) and (19) across all i yields the two equations

$$N_{t+1} = [1 - \theta] q_{d,t+1} \int K_{i,t+2}^d di + [1 - \theta] q_{nd,t+1} \int K_{i,t+2}^{nd} di$$
 (A11)

$$K_{t+2}^d + K_{t+2}^{nd} = \int K_{i,t+2}^d di + \int K_{i,t+2}^{nd} di, \tag{A12}$$

where we have used  $\int N_{i,t+1} di = N_{t+1}$  and  $\int z_{i,t+2} di = 1$ . Given that the constraints of all entrepreneurs are binding, the budget constraint (A8) also holds at the aggregate level, i.e.,

$$N_{t+1} = [1 - \theta] q_{d,t+1} K_{t+2}^d + [1 - \theta] q_{nd,t+1} K_{t+2}^{nd}.$$

Together with the above system, this implies  $\int K_{i,t+2}^d di = K_{t+2}^d$  and  $\int K_{i,t+2}^{nd} di = K_{t+2}^{nd}$ . Therefore, the claim is proved.

In summary, we have proved that the equilibrium prices and quantities constructed in Proposition 1 satisfy the household's and entrepreneur's optimality conditions, and that the quantities satisfy market clearing conditions.

Finally, we provide a recursive relationship that can be used to solve for  $\theta(A, n)$  given the equilibrium constructed in Proposition 1. The recursion (8) implies

$$\mu_{t}N_{i,t} + \theta_{t}z_{i,t+1} \left(K_{t}^{d} + K_{t}^{nd}\right) = E_{t}M_{t+1} \left[ (1 - \lambda) \left( \mu_{t+1}N_{i,t+1} + \theta_{t+1} \left(K_{t+1}^{d} + K_{t+1}^{nd}\right) z_{i,t+2} \right) + \lambda N_{i,t+1} \right],$$

$$= E_{t}M_{t+1} \left[ \left\{ (1 - \lambda) \mu_{t+1} + \lambda \right\} N_{i,t+1} \right]$$

$$+ (1 - \lambda) z_{i,t+1} E_{t} \left[ M_{t+1}\theta_{t+1} \left(K_{t+1}^{d} + K_{t+1}^{nd}\right) \right]. \tag{A13}$$

Below, we first focus on simplifying the term  $E_t M_{t+1} \left[ \left\{ (1-\lambda) \mu_{t+1} + \lambda \right\} N_{i,t+1} \right]$ . Note that a binding collateral constraint together with the entrepreneur's budget constraint (4) implies

$$[1 - \theta] q_{d,t} K_{i,t+1}^d + [1 - \theta] q_{nd,t} K_{i,t+1}^{nd} = N_{i,t}.$$
(A14)

Equation (A14) together with the optimality condition (19) determine  $K_{i,t+1}^d$  and  $K_{i,t+1}^{nd}$  as functions of  $N_{i,t}$  and  $z_{i,t+1}$ :

$$K_{i,t+1}^{d} = \frac{[1-\theta] q_{nd,t} z_{i,t+1} \left( K_{t+1}^{d} + K_{t+1}^{nd} \right) - N_{i,t}}{[1-\theta] q_{nd,t} - [1-\theta] q_{d,t}},$$

$$K_{i,t+1}^{nd} = \frac{N_{i,t} - [1-\theta] q_{d,t} z_{i,t+1} \left( K_{t+1}^{d} + K_{t+1}^{nd} \right)}{[1-\theta] q_{nd,t} - [1-\theta] q_{d,t}}.$$
(A15)

Using Equation (A15) and the law of motion of net worth (9), we can represent  $N_{i,t+1}$  as a

linear function of  $N_{i,t}$  and  $z_{i,t+1}$ :

$$N_{i,t+1} = z_{i,t+1} \alpha A_{t+1} \left( K_{t+1}^d + K_{t+1}^{nd} \right) + (1 - \delta_d) q_{d,t+1} \frac{(1 - \theta) q_{nd,t} z_{i,t+1} \left( K_{t+1}^d + K_{t+1}^{nd} \right) - N_{i,t}}{(1 - \theta) (q_{nd,t} - q_{d,t})}$$

$$+ (1 - \delta_{nd}) q_{nd,t+1} \frac{N_{i,t} - (1 - \theta) q_{d,t} z_{i,t+1} \left( K_{t+1}^d + K_{t+1}^{nd} \right)}{(1 - \theta) (q_{nd,t} - q_{d,t})}$$

$$- R_{f,t} \theta \left( 1 - \delta_d \right) q_{d,t} \frac{(1 - \theta) q_{nd,t} z_{i,t+1} \left( K_{t+1}^d + K_{t+1}^{nd} \right) - N_{i,t}}{(1 - \theta) (q_{nd,t} - q_{d,t})}$$

$$- R_{f,t} \theta \left( 1 - \delta_{nd} \right) q_{nd,t} \frac{N_{i,t} - (1 - \theta) q_{d,t} z_{i,t+1} \left( K_{t+1}^d + K_{t+1}^{nd} \right)}{(1 - \theta) (q_{nd,t} - q_{d,t})} .$$

Because we are only interested in the coefficients on  $z_{i,t+1}$ , collecting the terms that involves  $z_{i,t+1}$  on both sides of (A13), we have:

$$\theta_t z_{i,t+1} \left( K_{t+1}^d + K_{t+1}^{nd} \right) = z_{i,t+1} \left( K_{t+1}^d + K_{t+1}^{nd} \right) \times Term,$$

where

$$Term = E_{t} \begin{bmatrix} \tilde{M}_{t+1} & \alpha A_{t+1} + (1 - \delta_{d}) q_{d,t+1} \frac{q_{nd,t}}{q_{nd,t} - q_{d,t}} \\ + (1 - \delta_{nd}) q_{nd,t+1} \frac{-q_{d,t}}{q_{nd,t} - q_{d,t}} \\ -R_{f,t} \theta q_{d,t} \frac{q_{nd,t}}{q_{nd,t} - q_{d,t}} \\ -R_{f,t} \theta q_{nd,t} \frac{-q_{d,t}}{q_{nd,t} - q_{d,t}} \end{bmatrix} + (1 - \lambda) E_{t} [M_{t+1} \theta_{t+1}].$$

We can simplify the first term using the first order conditions (29)-(31) to get

$$E_{t}\left[\tilde{M}_{t+1}\left\{\alpha\left(1-\nu\right)A_{t+1}\right\}\right].$$

Therefore, we have the following recursive relationship for  $\theta(A, n)$ :

$$\theta(A, n) = [1 - \delta + i(A, n)] \{\alpha(1 - \nu) E[M' \{\lambda + (1 - \lambda) \mu(A', n')\} A'] + (1 - \lambda) E[M' \theta(A', n')] \}.$$
(A16)

The term  $\alpha(1-\nu)A'$  is the profit for the firm due to decreasing return to scale. Clearly,  $\theta(A, n)$  has the interpretation of the present value of profit. In the case of constant returns to scale,  $\theta(A, n) = 0$ .

# **Appendix B: Data Construction**

This section describes how we (i) construct firm samples for empirical analysis and (ii) construct firm characteristics to control for fundamentals.

# B.1. Asset Prices and Accounting Data

Our sample consists of firms in the intersection of Compustat and CRSP (Center for Research in Security Prices). We obtain accounting data from Compustat and stock returns data from CRSP. Our sample firms include those with positive durability data and non-missing SIC codes and those with domestic common shares (SHRCD = 10 and 11) trading on NYSE, AMEX, and NASDAQ, except utility firms that have four-digit standard industrial classification (SIC) codes between 4900 and 4999, finance firms that have SIC codes between 6000 and 6999 (finance, insurance, trusts, and real estate sectors), and public administrative firms that have SIC codes between 9000 and 9999. We follow Campello and Giambona (2013) by excluding firm-year observations for which the value of total assets or sales is less than \$ 1 million. Following Fama and French (1993), we further drop closed-end funds, trusts, American Depository Receipts, Real Estate Investment Trusts, and units of beneficial interest. To mitigate backfilling bias, firms in our sample must be listed on Compustat for two years before including them in our sample. Macroeconomic data are from the Federal Reserve Economic Data (FRED) maintained by Federal Reserve in St. Louis.

# Appendix C: Additional Empirical Evidence

In this section, we provide additional empirical evidence on the relation of the asset durability and other firm characteristics and document the summary statistics of the asset durability across industries.

#### C.1. More Detailed Firm Characteristics

Table C.1 documents how differences in asset durability among firms are related to other firm characteristics. We report average durability and these characteristics across five portfolios sorted on the firm-level asset durability among financially constrained firms

[Place Table C.1 about here]

Generally speaking, our sample contains 1,821 firms. Five portfolios sorted on asset durability from the lowest to the highest quintile are evenly distributed, with the average number of firms ranging from 301 to 417. The cross-sectional variations in durability are large, ranging from 7.69 to 18 across five portfolios sorted on durability. Size does not vary a lot but presents a hump-shaped pattern across five portfolios. Moreover, a firm with a lower asset durability has a lower book-to-market ratio (B/M) and a higher investment rate (I/K) and Tobin's q to reflect more investment opportunities. We also notice that low durability firms are less profitable, as measure of return on assets (ROA), and lower capacity to borrow, as measure by book leverage, and more financially constrained (SA and WW index). These characteristics suggest an endogenous choice for less durable assets when a firm becomes more financially constrained with low tangibility but faces a positive investment opportunity. Finally, there is a negative relationship between asset durability and collateralizability.

# C.2. Summary Statistics across Industries

In Table C.2, we report the average of asset durability and depreciation with respect to tangible and intangible assets in each industry according to the BEA industry classifications. Asset durability (depreciation) in some industries are higher (lower), such as the educational services and the accommodation industry. There are comparatively large cross-industry variations in asset durability (depreciation), ranging from 10.84 to 49.49. Therefore, to make sure our results are not driven by any particular industry, we control for industry effects as detailed later.

[Place Table C.2 about here]

#### Table C.1: Firm Characteristics

This table reports time-series averages of the cross-sectional median of firm characteristics in five portfolios sorted on asset durability, relative to their industry peers, where we use the NAICS 3-digit classifications and rebalance portfolios at the end of every June. The sample is from 1977 to 2016 and excludes financial, utility, and public administrative industries from the analysis. We split the whole sample into financially constrained and unconstrained firms at the end of every June, as classified by dividend payment dummy (DIV) according to Farre-Mensa and Ljungqvist (2016), and report five portfolios across the financially constrained subsample. The detailed definition of the variables is listed in C.3.

| Variables           | L     | 2     | 3     | 4     | Н     |
|---------------------|-------|-------|-------|-------|-------|
| Asset Durability    | 7.69  | 9.99  | 11.45 | 14.24 | 18.00 |
| Depreciation        | 0.19  | 0.16  | 0.15  | 0.13  | 0.11  |
| Log ME              | 4.88  | 5.13  | 5.16  | 5.22  | 5.07  |
| $\mathrm{B/M}$      | 0.48  | 0.51  | 0.53  | 0.60  | 0.67  |
| I/K                 | 0.37  | 0.30  | 0.28  | 0.24  | 0.22  |
| q                   | 1.65  | 1.54  | 1.48  | 1.37  | 1.27  |
| ROA                 | 0.07  | 0.09  | 0.10  | 0.11  | 0.11  |
| ROE                 | 0.12  | 0.17  | 0.18  | 0.22  | 0.23  |
| OC/AT               | 0.36  | 0.25  | 0.21  | 0.17  | 0.13  |
| R&D/AT              | 0.03  | 0.03  | 0.03  | 0.00  | 0.00  |
| Collateralizability | 0.21  | 0.25  | 0.27  | 0.37  | 0.51  |
| Book Lev.           | 0.13  | 0.19  | 0.21  | 0.28  | 0.32  |
| Short-term Lev.     | 0.02  | 0.02  | 0.02  | 0.03  | 0.03  |
| Long-term Lev.      | 0.04  | 0.09  | 0.11  | 0.17  | 0.21  |
| TANT                | 0.08  | 0.13  | 0.17  | 0.25  | 0.34  |
| SA                  | -2.47 | -2.68 | -2.80 | -2.91 | -2.92 |
| WW                  | -0.16 | -0.18 | -0.19 | -0.20 | -0.20 |
| Cash Flow Duration  | 20.43 | 20.15 | 19.99 | 19.47 | 18.96 |
| Number of Firms     | 365   | 345   | 301   | 393   | 417   |

### Table C.2: Asset Durability and Depreciation across BEA Industries

This table reports summary statistics of the average asset durability and depreciation with respect to tangible and intangible assets across industries. Industries are based on BEA industry classifications. The sample period is 1977 to 2016.

| BEA Industries   | Tangible              |                | Intangible          |                |
|--|-----------------------|----------------|---------------------|----------------|
| Industry Name  | Durability            | Depreciation   | Durability          | Depreciation   |
| Farms  | 27.92                 | 0.07           | 2.58                | 0.40           |
| Forestry, fishing, and related activities                          | 24.43                 | 0.09           | 2.38                | 0.43           |
| Oil and gas extraction   | 14.98                 | 0.07           | 4.33                | 0.23           |
| Mining, except oil and gas   | 20.56                 | 0.07           | 4.50                | 0.23           |
| Support activities for mining                                      | 13.67                 | 0.09           | 3.40                | 0.30           |
| Utilities  | 40.49                 | 0.03           | 3.38                | 0.31           |
| Construction   | 20.13                 | 0.10           | 3.95                | 0.26           |
| Wood products  | 22.67                 | 0.07           | 4.61                | 0.23           |
| Nonmetallic mineral products                                       | 20.65                 | 0.07           | 5.90                | 0.17           |
| Primary metals   | 21.28                 | 0.07           | 5.73                | 0.17           |
| Fabricated metal products  | 19.36                 | 0.08           | 5.68                | 0.18           |
| Machinery  | 20.94                 | 0.07           | 5.68                | 0.18           |
| Computer and electronic products                                   | 22.97                 | 0.07           | 3.44                | 0.29           |
| Electrical equipment, appliances, and components                   | 23.98                 | 0.06           | 5.89                | 0.17           |
| Motor vehicles, bodies and trailers, and parts                     | 17.97                 | 0.08           | 3.19                | 0.31           |
| Other transportation equipment                                     | 24.09                 | 0.06           | 4.47                | 0.22           |
| Furniture and related products                                     | 23.05                 | 0.06           | 5.37                | 0.19           |
| Miscellaneous manufacturing  | 22.33                 | 0.07           | 5.86                | 0.17           |
| Food, beverage, and tobacco products                               | $21.90 \\ 22.65$      | 0.07           | 5.55                | $0.18 \\ 0.18$ |
| Textile mills and textile product mills                            |                       | 0.06           | 5.46                |                |
| Apparel and leather and allied products                            | 26.52                 | 0.06           | 5.73                | 0.17           |
| Paper products   | 18.12                 | 0.08           | 5.38                | 0.19           |
| Printing and related support activities                            | 19.06                 | 0.08           | 5.02                | 0.21           |
| Petroleum and coal products  | 21.09                 | 0.07           | 5.86                | 0.17           |
| Chemical products Plastics and rubber products                     | 22.25                 | 0.07           | 8.09                | 0.12           |
| 1  | 18.44                 | 0.08           | 5.72                | 0.18           |
| Wholesale trade  | 24.93                 | 0.08           | 4.13                | 0.25           |
| Retail trade   | 33.63                 | 0.05           | 4.05                | 0.26           |
| Air transportation   | 19.23 $44.31$         | $0.07 \\ 0.03$ | $3.28 \\ 4.30$      | $0.31 \\ 0.25$ |
| Railroad transportation Water transportation                       | 18.99                 | 0.06           | 4.08                | 0.25           |
|  | 11.49                 | 0.06           | 4.19                | 0.26           |
| Truck transportation Transit and ground passenger transportation   | 35.17                 | 0.14           | 3.50                | 0.26           |
| Pipeline transportation  | 39.5                  | 0.03           | 3.12                | 0.30 $0.32$    |
| Other transportation and support activities                        | 30.07                 | 0.06           | 3.50                | 0.32           |
| Warehousing and storage  | 37.45                 | 0.04           | 3.88                | 0.28           |
| Publishing industries (including software)                         | 23.51                 | 0.04           | 6.39                | 0.16           |
| Motion picture and sound recording industries                      | 29.43                 | 0.05           | 7.86                | 0.13           |
| Broadcasting and telecommunications                                | 34.89                 | 0.04           | 5.42                | 0.19           |
| Information and data processing services                           | 22.86                 | 0.10           | 4.50                | 0.23           |
| Federal Reserve banks  | 34.66                 | 0.05           | 3.25                | 0.23           |
| Credit intermediation and related activities                       | 26.75                 | 0.07           | 2.99                | 0.34           |
| Securities, commodity contracts, and investments                   | 35.37                 | 0.04           | 3.12                | 0.32           |
| Insurance carriers and related activities                          | 33.83                 | 0.05           | 3.10                | 0.33           |
| Funds, trusts, and other financial vehicles                        | 40.54                 | 0.03           | 3.02                | 0.33           |
| Real estate  | 40.04                 | 0.03           | 2.89                | 0.35           |
| Rental and leasing services and lessors of intangible assets       | 10.84                 | 0.12           | 2.87                | 0.35           |
| Legal services   | 31.14                 | 0.06           | 2.57                | 0.40           |
| Computer systems design and related services                       | 31.76                 | 0.07           | 2.83                | 0.35           |
| Miscellaneous professional, scientific, and technical services     | 26.62                 | 0.07           | 5.41                | 0.19           |
| Management of companies and enterprises                            | 35.71                 | 0.04           | 3.23                | 0.31           |
| Administrative and support services                                | 29.09                 | 0.07           | 2.79                | 0.36           |
| Waste management and remediation services                          | 48.14                 | 0.05           | 3.91                | 0.26           |
| Educational services   | 49.49                 | 0.03           | 4.80                | 0.20           |
| Ambulatory health care services                                    | 34.39                 | 0.06           | 4.86                | 0.21           |
| Hospitals  | 45.77                 | 0.04           | 4.39                | 0.21 $0.24$    |
| Nursing and residential care facilities                            | 39.67                 | 0.04           | 5.05                | 0.24           |
| Social assistance  | 37.26                 | 0.04           | 3.18                | 0.20 $0.32$    |
| Performing arts, spectator sports, museums, and related activities | 36.87                 | 0.04           | 6.10                | 0.16           |
|  | 30.35                 | 0.04           | 3.95                | 0.16           |
| Amusements, gambling, and recreation industries Accommodation  67  | 48.59                 | 0.03           | $\frac{3.93}{4.07}$ | 0.25           |
| Food services and drinking places                                  | $\frac{46.59}{27.15}$ | 0.03           | 4.16                | $0.25 \\ 0.24$ |
| Other services, except government                                  | 43.02                 | 0.04           | 5.24                | 0.19           |

Table C.3: **Definition of Variables** 

| Variables          | riables Definition   |                 |  |
|--------------------|--|-----------------|--|
| Durability         | Details refer to Section 2.1   | BEA; Compustat  |  |
| Depreciation       | Details refer to Section 2.1   | BEA; Compustat  |  |
| ME (real)          | Market capitalization deflated by CPI at the end of June in year t.  | CRSP            |  |
| B/M                | The ratio of book equity of fiscal year ending in year t-1 to market equity at the end of year t-1.  | Compustat       |  |
| Tobin's q          | The sum of market capitalization at the end of year and book value of preferred shares deducting inventories over total assets (AT).   | CRSP; Compustat |  |
| I/K                | The ratio of investment (CAPX) to purchased capital (PPENT).   | Compustat       |  |
| ROA                | The ratio of operating income before depreciation (OIBDP) over total assets (AT).  | Compustat       |  |
| ROE                | The ratio of operating income before depreciation (OIBDP) over book equity.  | Compustat       |  |
| OC/AT              | Following Peters and Taylor (2017).  | Compustat       |  |
| R&D Intensity      | Following Peters and Taylor (2017).  | Compustat       |  |
| Tangibility        | The ratio of purchased capital (PPENT) to total assets (AT).   | Compustat       |  |
| Book Lev.          | The sum of long-term liability (DLTT) and current liability (DLCT) divided by total assets (AT).   | Compustat       |  |
| Short-term Lev.    | Current liability (DLCT) divided by total assets (AT).   | Compustat       |  |
| Long-term Lev.     | Long-term liability (DLTT) divided by total assets (AT).   | Compustat       |  |
| DIV                | Following Farre-Mensa and Ljungqvist (2016).   | Compustat       |  |
| SA Index           | Following Hadlock and Pierce (2010).   | Compustat       |  |
| Credit Rating      | The entire list of credit ratings is as follows: AA+, AA, and AA- = 6, A+, A, and A- = 5, BBB+, BBB, BBB- = 4, BB+, BB, BB- = 3, B+, B, and B- = 2, rating below B- or missing is 0. | Compustat       |  |
| WW Index           | Following Whited and Wu (2006).  | Compustat       |  |
| Cash Flow Duration | Following Dechow et al. (2004).  | Compustat       |  |